Teaching Mathematics for Robust Understanding
What makes a mathematically powerful classroom?

Leader's Guide

GOAL

This tool is intended to help develop an understanding of the Teaching for Robust Understanding (TRU) framework for Mathematics as a foundation for thinking about, planning, observing, and reflecting on classroom teaching.

USERS

This introduction can be used with a wide variety of groups, including: district personnel who are in a position to coach, observe, or otherwise support teacher professional development; principals who need to provide a supportive environment for staff learning; in-service teachers at the beginning of professional development; tutors and student teachers in the early stages of a teacher preparation course. It provides a compact introduction to the suite of materials on http://map.mathshell.org/trumath.php

INTRODUCTION

Reflecting the standards in high-performing countries, the Common Core and related state standards embody a broader and deeper view of mathematics than has been traditional in US schools. As well as emphasizing robust understanding of mathematical *content*, it includes the *practices* of doing mathematics. This requires a deeper way of thinking about what matters in classrooms.

Teaching for Robust Understanding in Mathematics (TRU) is a research-based framework for doing this. The five dimensions of TRU are: (i) the mathematics, (ii) cognitive demand, (iii) access to mathematical content, (iv) agency, ownership and identity and (v) formative assessment. If things go well in classrooms along these five dimensions, the students who emerge will be powerful thinkers and problem solvers.

The purpose of this workshop session is to introduce participants to the framework and to some of the tools that can be used with it. By the end of the workshop we hope they will see that:

- TRU is a way of thinking, not just a set of tools. It can be used by individuals and communities in an ongoing way for planning, observing, and reflection.
- Getting better across the dimensions of TRU leads to improved instruction.
- There are no "magic bullets." Change takes time, and TRU can help to support ongoing improvement.

The approach taken in this workshop is aimed at having participants see TRU emerge from their own thoughts about and discussions of specific examples of teaching. It gives participants a chance to react to the three stimulus videos, so that the analysis that follows is seen as pulling together their own thoughts, not imposing something from "the outside."

SESSION OUTLINE

- What matters in classrooms? — 5 minutes
- Part 1: An introduction to TRU – 3 videos analyzed — 15+15 + 10 + 10 = 50 minutes
- Part 2: Introducing TRU Tools — 20 minutes
- Q & A — 15 minutes

MATERIALS REQUIRED

- PowerPoint: TRU Slides.pptx
- Either a whiteboard large enough to record 5 columns' worth of participants' comments – or five flip charts, with markers. (See Slide 7 below)
- Session Handouts: One copy per participant.

TIME NEEDED

90 minutes, with opportunities for expansion.

PREPARATION

Please work through this guide carefully, referring to the Handouts and Powerpoint. The core Activity Sequence (below) covers the same material as on the PowerPoint slides, including the notes with each slide, distinguishing suggested comments and *instructions*. Fill in your local information on the first and last slides.

Part 1 is intended to go slowly. Participants will generate the ideas that you, the leader, synthesize. This part is interactive. Be sure that you get comfortable with TRU yourself so you can recognize which of its dimensions each comment from the participants in Part 1 fits best.

Part 2 is more of a presentation, where three tools are presented. The idea is to zip through things, taking time to linger on just one page as an example.

Try to anticipate the common issues that participants will have and note down your responses to them, in the table below. The examples shown were taken from trials of this session.

Common concern	Suggested responses
• It seems awfully complicated, with it's five dimensions. I thought teaching was about explaining the math, showing how each procedure works, and having the students practice until they know it.	• That's a traditional approach and it works with some students - but you know that a lot of students, even those who 'get it' at the time, can't do it six months later. That's why TRU is about "*robust understanding*" - which depends on students geting to grips with the math more deeply (cognitive demand) and making it their own not just the teacher's (agency, ownership, identity)
• Doesn't teaching like this take a lot more time?	• It does, but it saves a lot of re-teaching topics - and re-teaching, usually faster, is not an effective approach to remediation.
•	•
•	•

ACTIVITY SEQUENCE

Title Slide

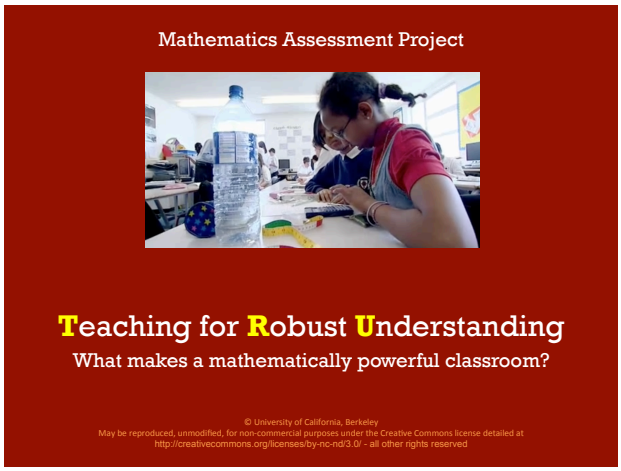
You may like to customize this slide and/or the last one with your own institutional and contact details. Please leave the copyright attribution, however.

Possible comments below are in plain text. *Suggestions are in italics.*

Users will, of course, adapt as necessary – though we recommend sticking with this activity sequence the first time or two.

Introduce the session by saying (in your own words) that the purpose of the session is to provide participants with a deep sense of the Teaching for Robust Understanding (TRU) framework.

Slide 1



Introduction (5 minutes)

In Part 1 we develop a framework that seeks to answer the question: What really matters in classrooms?

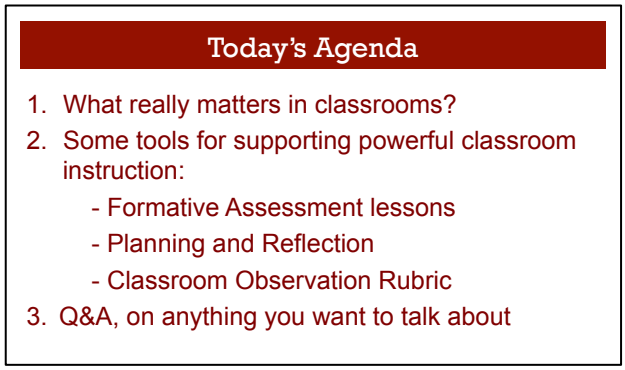
We will do this by watching three videos and discussing what what the lesson is trying to achieve.

The framework will emerge from your conversations.

In part 2 I'll introduce some tools that help to support powerful classroom instruction.

Then we'll close with questions and answers.

Slide 2



1. What matters in classrooms?

Show this slide quickly, then move on.

Slide 3

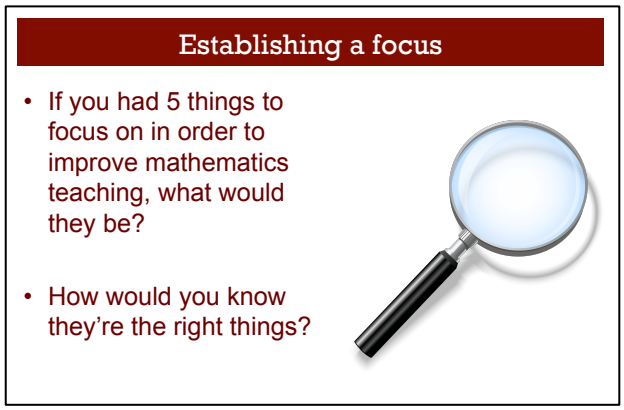
Give people a bit of time to discuss and note down answers to these two questions.

Note your ideas on **Handout 1.**

You may well ask: why just five?

- It's as many as most folks can keep in mind. (In fact, it may be too many to work on at one time.)
- If you have 20, you might as well have none. People can't keep that many things in their heads, and long checklists don't help. What matters is what people can act on, in teaching and coaching.

Slide 4



1. The angles lesson
(~15 minutes after 5 minutes)

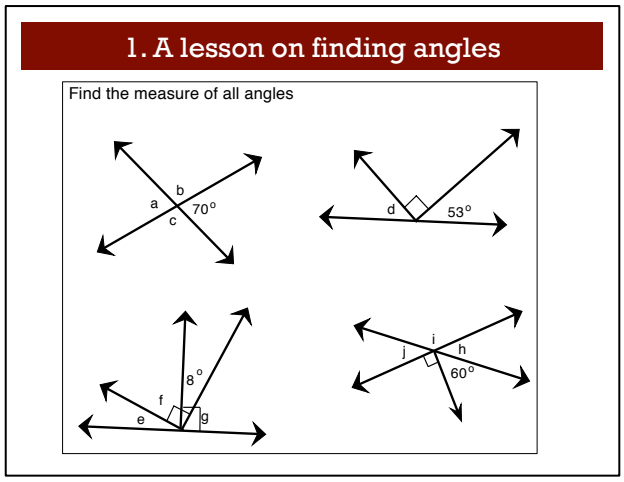
We'll now talk about what you see in each of three videos. The TRU framework will emerge from your observations.

Feel free to compare and contrast – but make sure to focus on what the experience looks like from the point of view of the students.

The first lesson we'll look at is the TIMSS Geometry Video on Angles.

This is the task that students are looking at. Think about what it involves.

Slide 5



On your own, note down what you notice about the lesson on **Handout 2**

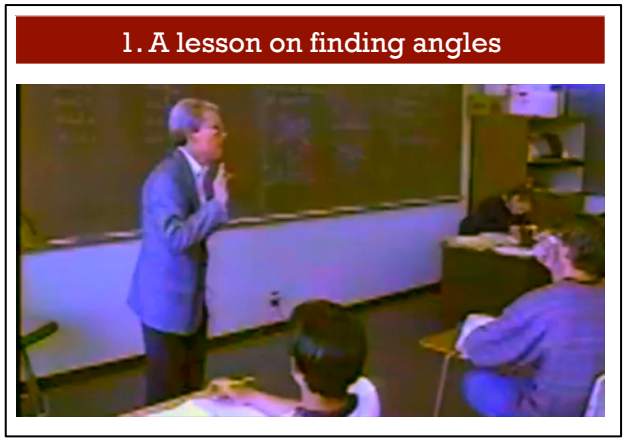
What is it like to be a student in this classroom?

Play the video. (4 minutes).

This video is taken from:

http://www.timssvideo.com/videos/mathematics/United%20States

Slide 6



You need to mark out 5 columns on the whiteboard, (or have 5 flip charts set up).

Complete your notes on **Handout 2**

Then share your observations with a partner.

Give the group a minute to think then 1-2 minutes to talk in 2s or 3s.

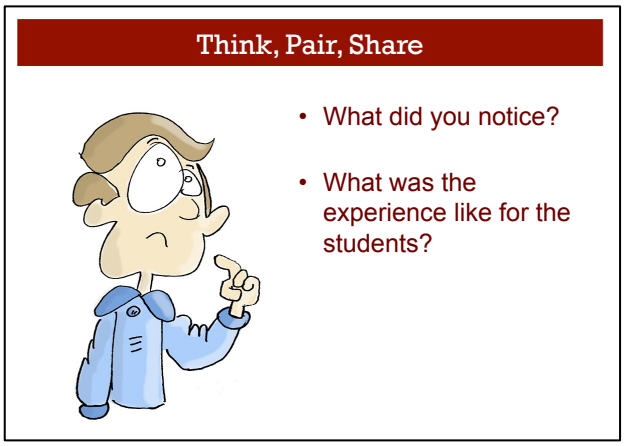
Now collect a few comments on the whiteboard or the flipcharts.

Have the 5 TRU dimensions in your head, but do not write them on the whiteboard yet. Leave room at the top for putting the headings after you've looked at all three videos.

*When a comment about **mathematics** is made, write it in the first column. Comments about **cognitive demand** go in column 2, **equity** column 3, **agency and identity** column 4, and **formative assessment** column 5. **Don't worry aboput being too accurate – it all works out in the end.***

Don't say anything about why; just move to the second video after a few minutes.

Slide 7



Vocabulary and small calculations of angles	Demand is low. Math is in bite-sized pieces. No thinking time.	Participation is limited – not much to think about.	No discussion, students sit apart.	Understanding is not revealed. Questioning is closed, and doesn't require explanations.

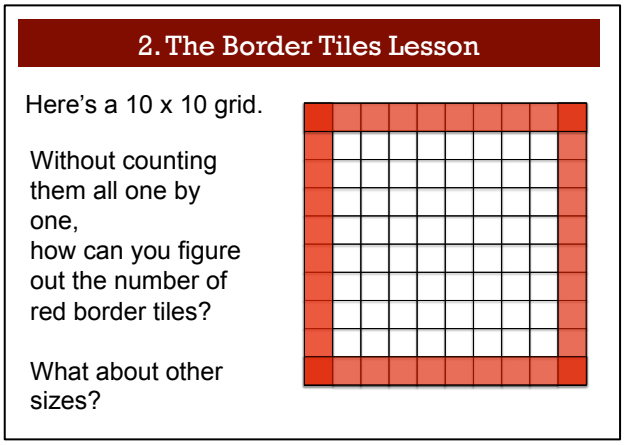
2. The border tiles lesson
(15 minutes after 20 minutes)

The set-up is important here.

Explain the problem and remind the participants to think about what it looks like from the student's perspective.

Toward the end of the tape tell them that next steps include thinking about the same problem with a 6 x 6 square, and then an n x n square.

Slide 8



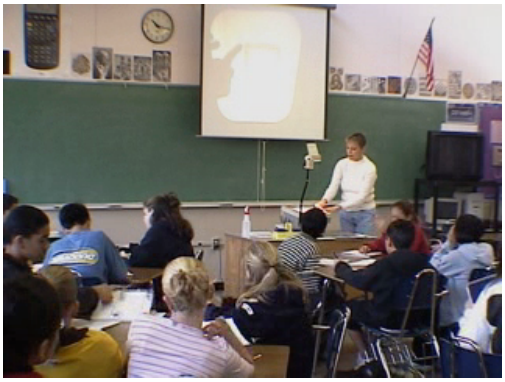
Again, make notes on what you see on Handout 2

Play the video (6 minutes)

From Boaler and Humphreys (2005)
Connecting Mathematical Ideas: Middle School
Video Cases to Support Teaching and Learning

Slide 9

2. The Border Tiles Lesson



Now we'll reflect on this lesson as we did before**.** *Write additional comments on the whiteboard using a different color.*

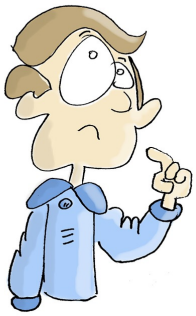
Work through enough comments to get the sense of how different this tape is from the first. Comparative comments are fine.

(Key points to keep in mind when facilitating the comments: the math starts out simple but gets interesting – will all the formulas give the same answer? Student thinking is the focus, and it's not trivial; there is some challenge. Lots of kids participate, and they're invted in; they get to talk math, and their thinking is fully heard.)

There is no need to say this yourself, but when you paraphrase as you write things down, these brief descriptions might be useful.

Slide 10

Think, Pair, Share



- What did you notice?
- What was the experience like for the students?

3. The fractions, decimals, percents lesson
(10 minutes after 35 minutes)

This lesson was with a 6th grade classroom in an inner-city, low income school in Chicago. The lesson uses materials from the project website:

http://map.mathshell.org/

It starts out with their cutting out the cards and filling in the blanks – they have to write in the % if the decimal is given, and vice-versa.

Then they have to order them.

It's essential that they collaborate.

Notice that one card has no numbers on it.

Slide 11

3. Fractions, decimals percents

Take turns to:

1. Fill in the missing decimals and percents.
2. Place the cards in order of size.
3. Check that you agree.

0.2 ___%	0.05 ___%	_.___ 80%
0.375 ___%	_.___ 12.5%	0.75 ___%
1.25 ___%	_.___ 50%	_.___ ___%

Move quickly through to the video – this is just to give a picture of the full lesson

In the full lesson, after they do decimals and percents, the students are given other representations and have to match the cards.

First they are given area diagrams.

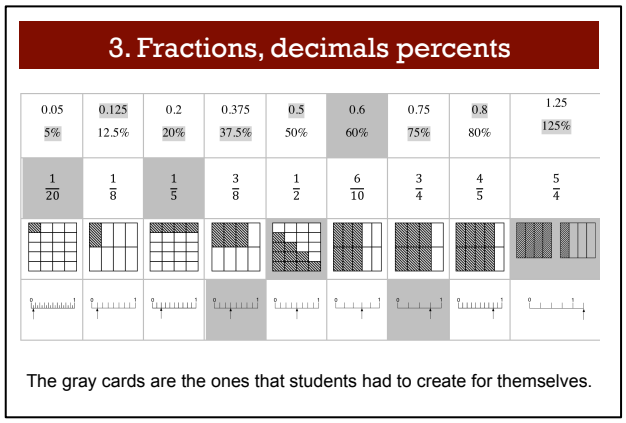
Then they are given fraction representations and scales.

As cards are placed, they are again ordered.

Students are encouraged to collaborate on this, detecting and correcting errors, and filling in the gaps.

The slide shows the answer set. THE gray squares indicate blank cards filled in by the students.

Slide 12

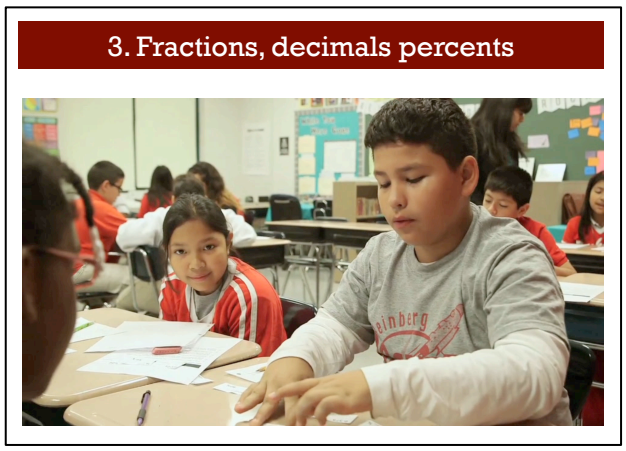


Now for a video episode in a class working on the first set of cards.

Again note what you observe on Handout 2

Play the video. (3 minutes)

Slide 13

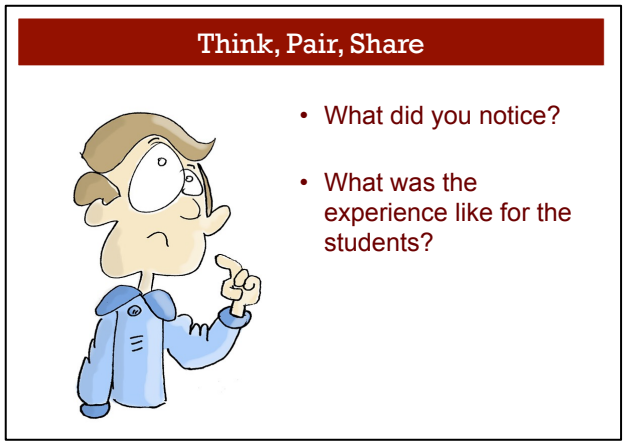


Again give time for people to complete their notes.

Now that this is the last video, give a little more time in the "share" portion for people to say what they want, including comparative comments.

You're still organizing them in 5 columns, without headings.

Slide 14



Recognizing TRU
(10 minutes after 45 minutes)

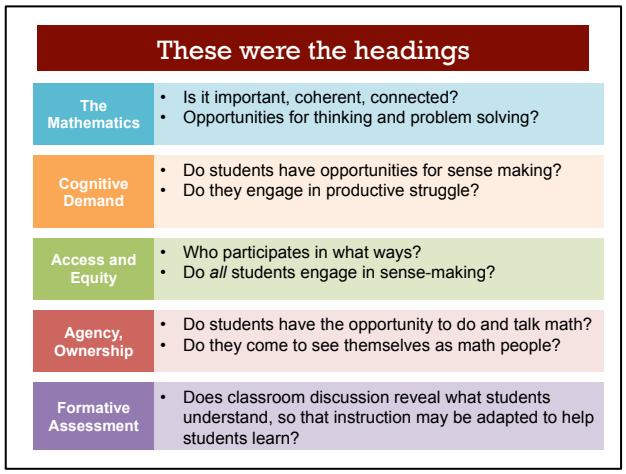
Now let me show you how I organized your comments.

At this point write the headings on top of each of the columns (or flip charts). As you do, read through the questions on the slide.

The slide shows a brief summary of the TRU framework. It's on **Handout 3**.

So what's new or different? In a sense, nothing. You should recognize and resonate to everything in TRU. It captures what we know is important. It doesn't offer any "magic bullets" or surprises.

Slide 15



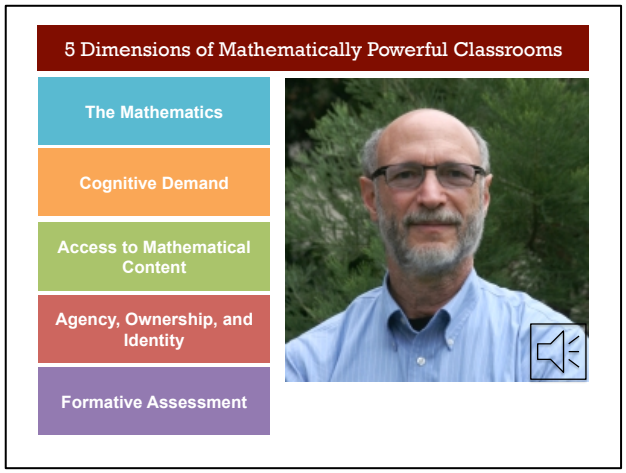
Alan Schoenfeld has led the team, who have been working for many years on TRU. As I play the audio file, Alan will introduce us to the framework.

As Alan speaks, try to relate the foci of your classroom observations to the various categories in his framework.

Click on the picture of Alan to hear the audio file. *(4 minutes)*

TRU Math was developed to highlight critical components of teaching and learning It provides a means of structuring our observations and discussions with teachers.

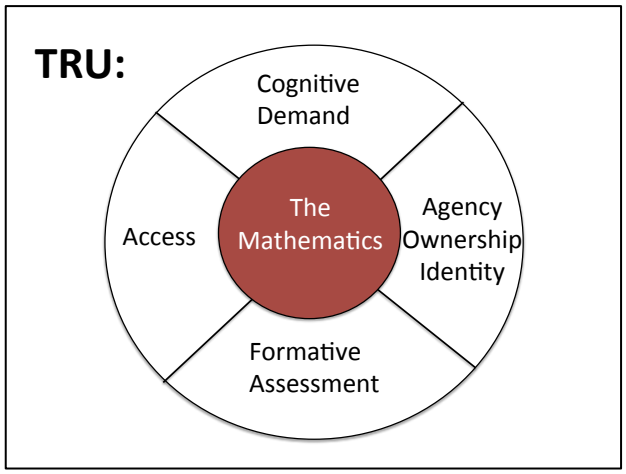
Slide 16



There's one problem with what you've seen thus far. Text is linear, but the ideas aren't. So here's a re-framing.

The math is at the heart of everything; and everything is connected, either by adjacency or across. For example, access to the math is a key ingredient on the way to developing agency, but you get it by working at the right level of cognitive demand ("productive struggle"), which is best set if you have a chance to hear what students think and adjust instruction accordingly (formative assessment). So, it's all connected, BUT, you can take it apart to work on any dimension you want to focus on.

Slide 17



Alan Schoenfeld and colleagues claim that
Read the slide

But before proceeding, it's ESSENTIAL to understand:

- TRU is NOT a tool or set of tools.
- TRU is a perspective regarding what counts in instruction, and
- TRU provides a language for talking about instruction in powerful ways.

With this understanding, you can make use of any productive tools more wisely.

Evidence for the claims may be found at http://map.mathshell.org *and* http://ats.berkeley.edu

Slide 18

Our claim

Research suggests that:

- Classrooms at all grade levels that do well along these five dimensions will produce students who are powerful mathematical thinkers.
- Instructional materials, professional development, and classroom observations will be most powerful if they are aligned with these five dimensions.

Part 2. Tools for supporting instruction
(20 minutes after 55 minutes)

This part – all of Part 2 – is in "show and tell" mode - in contrast to the workshop mode for Part 1.

From here on we'll briefly look at three types of tools.

Read the slide

More tools, and more information, are available at the websites (which will be shown again at the end of the workshop).

Slide 19

2. Tools for supporting powerful classroom instruction

a. Formative Assessment lessons
b. Planning and Reflection
c. Ways to observe classrooms

Tool 1: Formative Assessment Lessons

Classroom Challenges are lessons that support teachers in formative assessment. There are 100 lessons in total, 20 at each grade from 6 to 8 and 40 for 'Career and College Readiness' at High School Grades 9 and above. Some lessons are focused on developing math concepts, others on solving non-routine problems. They each take about 2-3 class periods.

The goal here is to to take a high speed tour through one FAL and show how well the FALs support teaching that's in line with TRU.

Slide 20

Tool (a) Formative Assessment Lessons

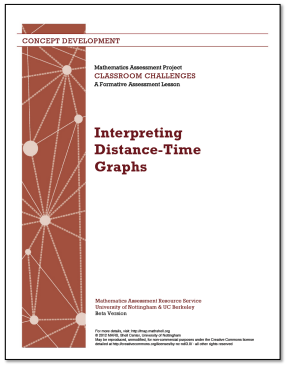
http://map.mathshell.org

100 Classroom Challenges:

- 20 at each grade 6-8
- 40 for Career and College readiness at High School.

- 70% concept focused
- 30% problem solving

Each lesson attempts to integrate formative assessment into everyday teaching.



FALs start with a diagnostic task, which is not graded. It gives the teacher a chance to see what the students understand (or don't). Many teachers are shocked to discover that roughly 1/3 of their students will write something like,

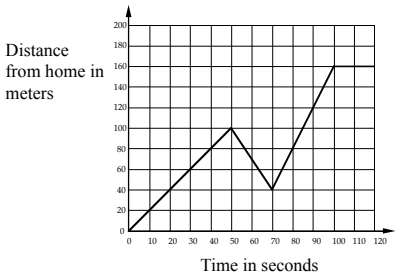
"Tom walked up a steep hill, got to the top and walked down for a while, until he had to walk up an even steeper hill, before he waited at the bus stop, which was flat."

It may not be pleasant to discover this, but it's essential to know what your students are thinking!

Slide 21

Diagnostic task before the lesson

Every morning Jane walks along a straight road to a bus stop 160 meters from her home, where she catches a bus to college.
The graph shows her journey on one particular day.
Describe what may have happened.
Is the graph realistic? Why?



This lesson has been taught many times, and researchers have studied what students do.

The lesson plan indicates what is likely to happen when students work on the lesson. It helps prepare the teacher for things the students will say, and makes suggestions for ways to respond without simply giving the answer – using questions.

You may want to read one or two of these, but do not spend a lot of time on them.

Slide 22

Common Issue	Possible questions and prompts
Student interprets the graph as a picture E.g. as the graph goes up and down, Tom's path goes up and down.	• If a person walked at a steady speed up and down a hill, *directly away from home*, what would the graph look like?
Student interprets graph as speed–time E.g. The student has interpreted a positive slope as speeding up and a negative slope as slowing down.	• How can you tell if Tom is traveling away from or towards home?
Student fails to mention distance or time E.g. The student has not worked out the speed of some/all sections of the journey.	• Can you provide more information about how far Tom has traveled during different sections of his journey?
Student fails to calculate and represent speed	• Can you provide information about Tom's speed for all sections of his journey?
Student adds little explanation as to why the graph is or is not realistic	• Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why?/Why not?

Now the teacher is prepared for the lesson.

The lesson begins with a whole class discussion using this task. Which is the best story?

About 1/3 of the students typically vote for each option.

Work through option B, which represents the "taking a graph to represent a picture of the situation" mistake.

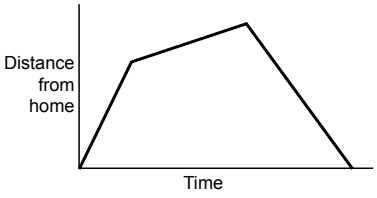
Slide 23

Lesson beginning: Which story best fits?

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

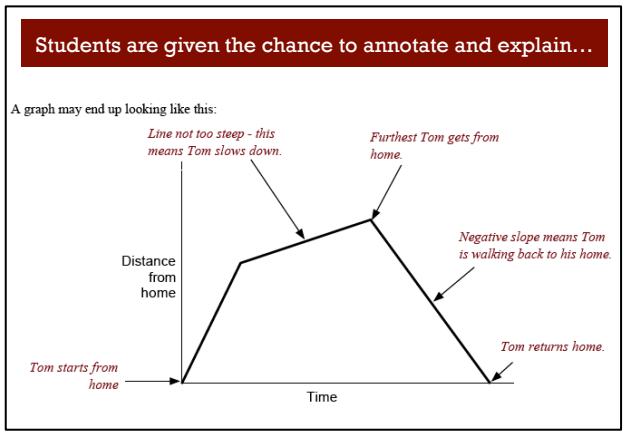
C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.



In the next segment of the lesson, teacher and class work through the graph in the previous slide, discussing what the different graph segments mean in terms of distance and time, and describing each one.

This part of the lesson serves to model the reasoning that the teacher expects of the students in the rest of the lesson.

Slide 24



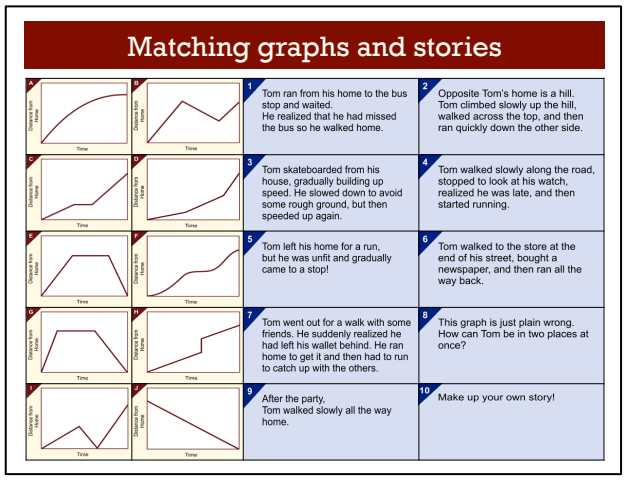
Students are now given a collection of graphs and stories on cards.

They are asked to work in groups to match stories to graphs, as they prepare to make posters.

The teacher can circulate through the classroom as they do. It's easy to see where the students are making sense, where they're being challenged.

The use of shared resources like this encourages discussion among students. The teacher can listen to their reasoning, and challenge it where necessary.

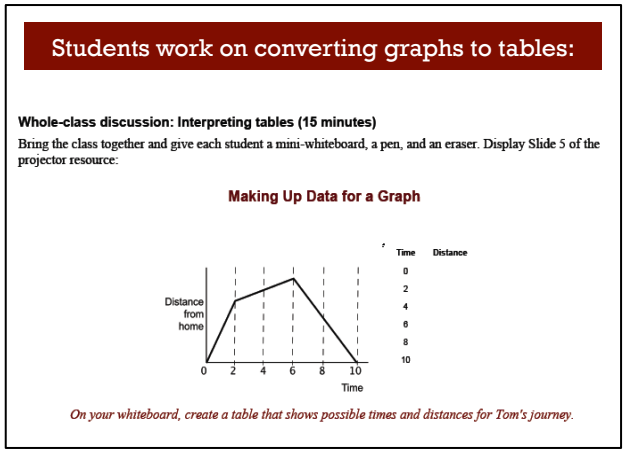
Slide 25



Now another whole class discussion. The teacher asks students to think again about the situation in a different way.

To help the students make connections, the teacher shows them how they can take one of the graphs – which does not have numbers on the axes – and make it more concrete, by assigning numbers. Then they can make a table using those numbers. The table helps to make sense of the stories that might match the graph.

Slide 26

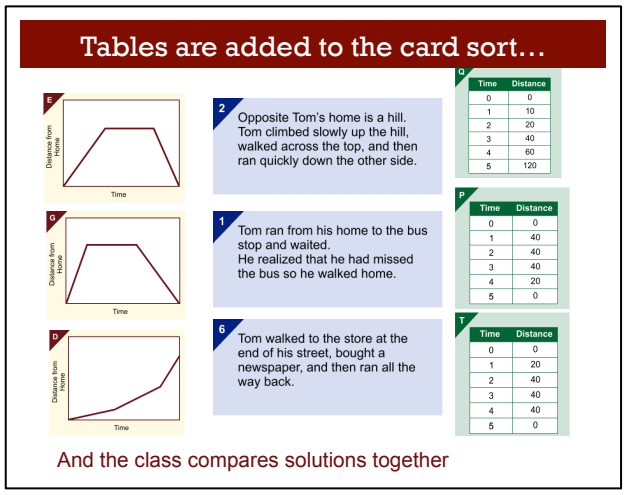


This new representation - the table – makes things more understandable, so we now ask the students to make posters that contain triples of matching graphs, stories, and tables.

The slide has 3 of each. You might like to spend a minute matching them, talking with your neighbour.

In the lesson, there are 10 of each

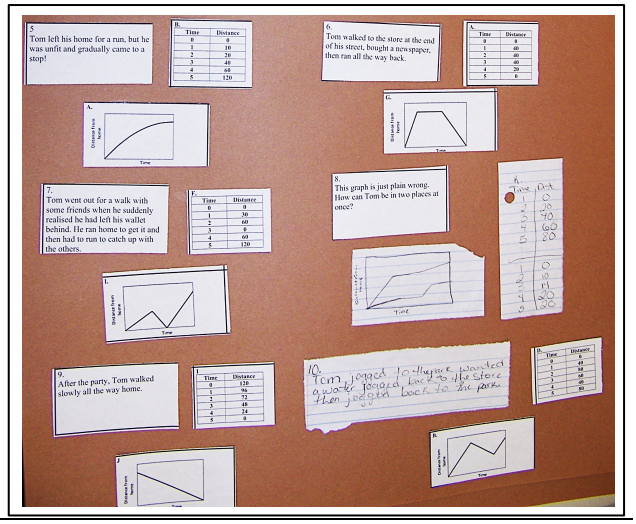
Slide 27



That yields posters like this one, which they then go on to annotate, explaining WHY each triad fits together.

The students compare posters with each other, and may present them to the whole class.

Slide 28

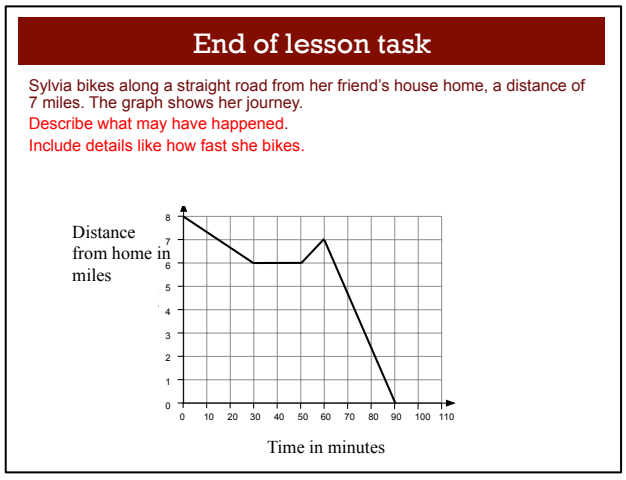


Finally, the lesson concludes with the teacher returning the original diagnostic task and asking the students to rethink it, correcting any earlier mistakes. Students are invited to state why they have changed their minds about anything.

They are also asked to do another, similar task.

In this way, both the student and teacher can monitor what has been learned.

Slide 29



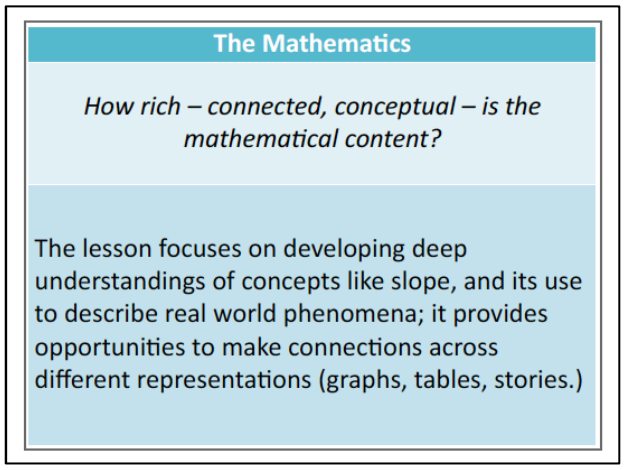
So how does this lesson match up to the TRU framework?

This slide shows how it matches the Mathematics Dimension.

Its clearly important math and the lesson makes many connections.

In what follows, you want to move quickly – naming each dimension and paraphrasing what's on the bottom of the slide. Make clear, on the run, that the FALs and TRU are completely in synch.

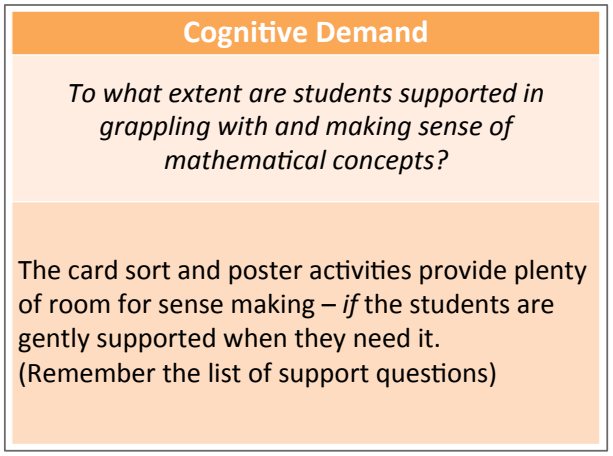
Slide 30



The lesson is also challenging. The whole lesson is about sense-making.

The teacher's role is to help students interpret and translate between the different representations.

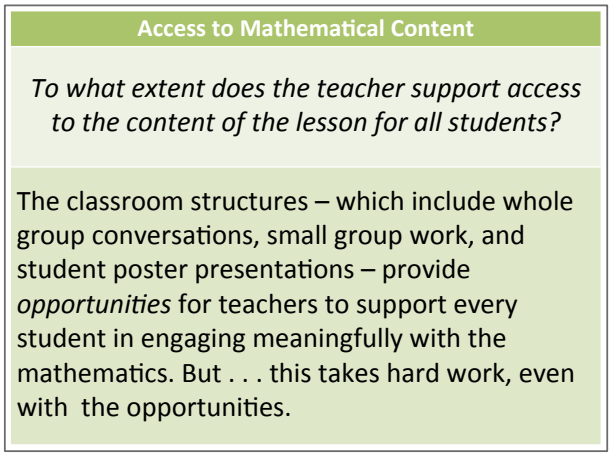
Slide 31



No curricular materials can do the complete job with regard to access and agency/identity – this takes hard, sensitive work on the part of the teacher.

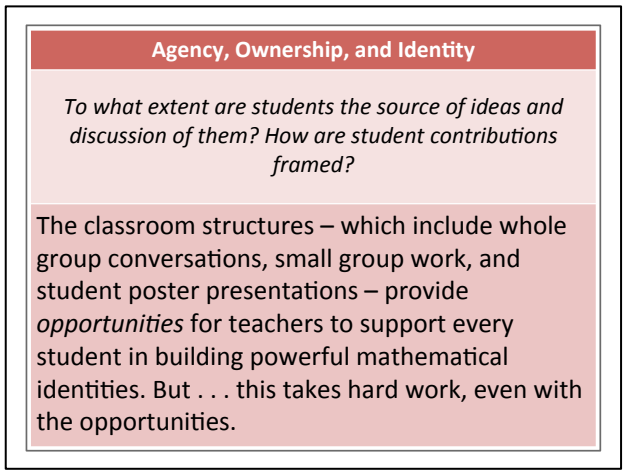
However, the shared resources and the lesson structure provides opportunities for all students to collaborate and reason together.

Slide 32



Again, this isn't guaranteed to happen. But if the teacher uses these materials in the ways promoted in the teacher guide, then students will be involved in explaining to one another and engaging in each other's reasoning.

Slide 33

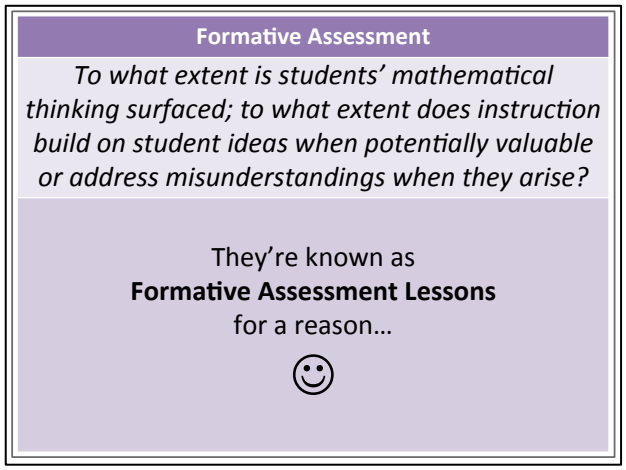


These are known as formative assessment lessons for a reason!

- The initial task exposes current thinking.
- The tasks are designed to promote discussion about common misconceptions.
- Students are invited to assess the reasoning of other students.
- This process moves students' reasoning forward – – the lesson provides diagnosis leading into treatment.

So you see, in all five dimensions, these lessons and TRU are in perfect harmony!

Slide 34



Now the question: so, everything fits together – but, does it work?

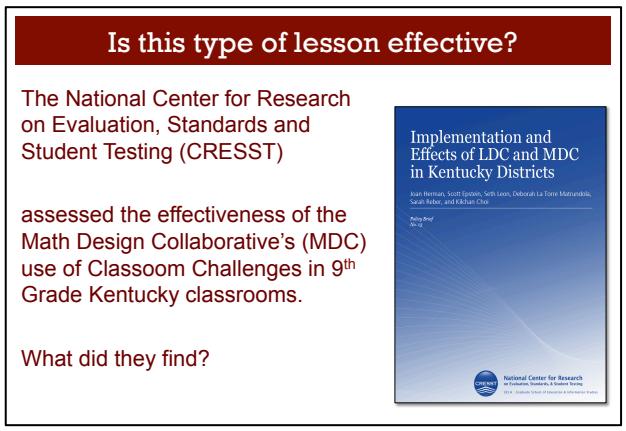
The Gates Foundation supported professional development in various places, including Kentucky.

Professional development was provided by the Mathematics Design Collaborative, which helped teachers implement these lessons in the ways the designers intended.

The Foundation hired independent evaluators to examine the impact of them.

Source:
https://www.cse.ucla.edu/products/policy/PB_13.pdf

Slide 35



Here is what they found.

That's almost unbelievable. How could it be?

What makes the lessons effective is that the teacher shifts to a much more student-centered pedagogy – "meeting students where they are" and helping them build up their understandings. This is a skill that transfers to regular instruction – and when it does, students learn more.

So, the impact of the lessons goes beyond them – teachers are moving to a "classroom culture" more in line with TRU.

Slide 36

Is this type of lesson effective?

- "For MDC, participating teachers were expected to implement between four and six Challenges, meaning that **students were engaged only 8-12 days of the school year**
- Nonetheless, the studies found **statistically significant learning effects… the approximate equivalent of 4.6 months for MDC**. Given their contexts of early implementation and limited dosage, these small effects are noteworthy."

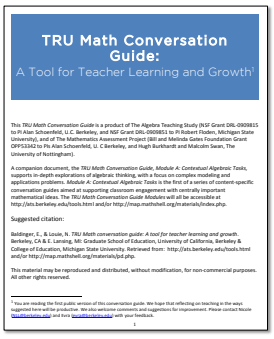
The next tool is designed to help teachers refine their teaching, more in line with TRU.

This tool is reproduced in full in **Handout 4.**

Slide 37

Tool (b) Planning and Reflection

- The TRU Math Conversation Guide.
- The TRU dimensions become arenas for teachers to reflect on their own teaching:
  - in planning,
  - in reflecting on how things have gone
  - in thinking about next steps.


TRU Math Conversation Guide:
A Tool for Teacher Learning and Growth

The idea is to take core questions related to the five dimensions…

Slide 38

Start with the core questions:

The Mathematics
How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?

Cognitive Demand
What opportunities do students have to make their own sense of mathematical ideas?

Access to Mathematical Content
Who does and does not participate in the mathematical work of the class, and how?

Agency, Ownership, and Identity
What opportunities do students have to explain their own and respond to each other's mathematical ideas?

Formative Assessment
What do we know about each student's current mathematical thinking, and how can we build on it?

And turn them into sets of questions that can be used for planning and reflection – perhaps by oneself, but even better with colleagues.

Slide 39

. . . and expand them.

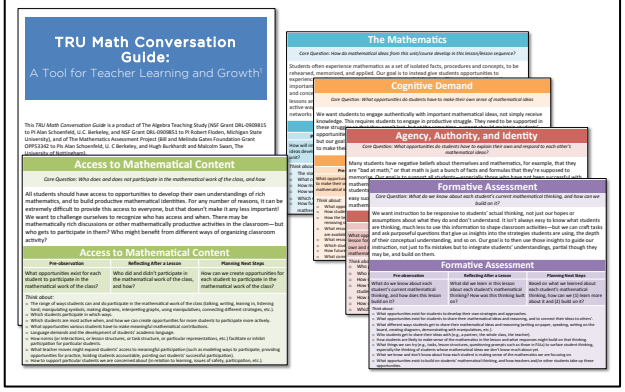
- **Before a lesson**, you can ask:
  - How can I use the five dimensions to enhance my lesson planning?
- **After a lesson**, you can ask:
  - How well did things go? What can I do better next time?
- **Planning next Steps**, you can ask:
  - How can I build on what I've learned?

Now refer participants to **Handout 4**

Looking at the TRU conversation guide, I'm going to flip through the guide to show you what it looks like, and make a quick stop at "access" to illustrate the kind of conversations its designed to support.

Slide 40

Looking at the conversation guide



This is what the expansion for access looks like. Let's take a closer look at this one. I'm going to read some of the "think abouts" down at the bottom.

"Think about:

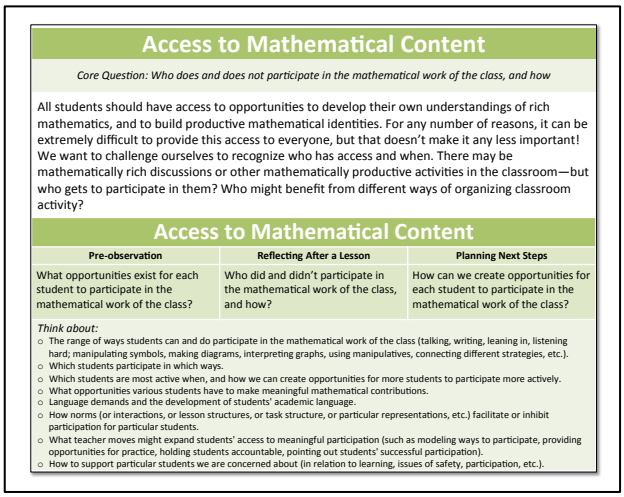
- The range of ways students can and do participate in the mathematical work of the class.
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions."

Now imagine teachers and coaches planning together, watching each other teach and debriefing using these ideas.

What's critically important is to make thinking like this a habit, so you think about these issues all the time – in planning, in teaching, in reflecting.

We've built distilled versions that are useful in watching videos in PD, or for keeping "at the top of your head" for reflection.

Slide 41

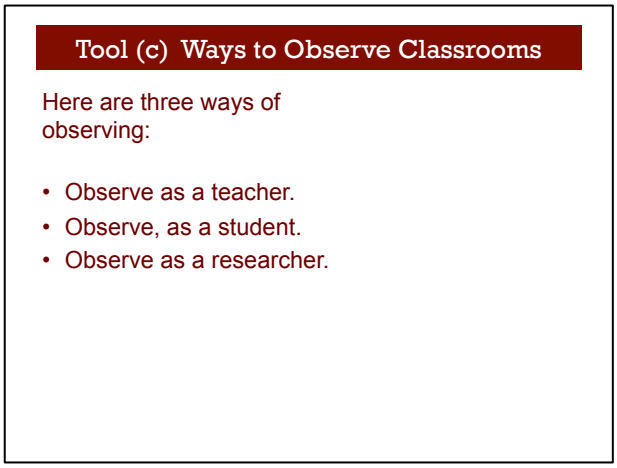


The third and final tool we will consider is the observation of classrooms: **Handout 5**

This can be done in many ways, depending on the purpose. However, TRU has been modified to suit these different purposes.

Lets look quickly at three.

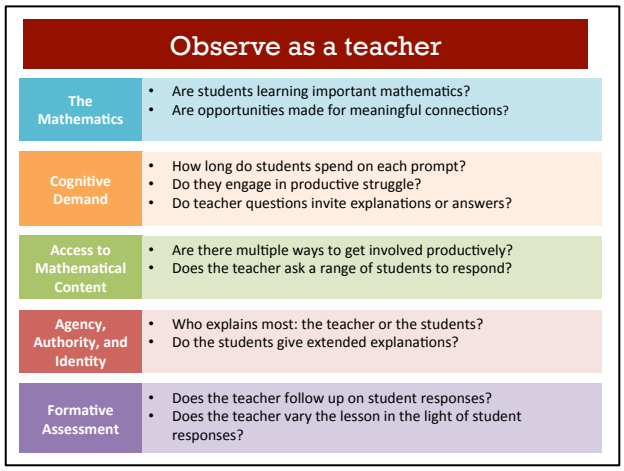
Slide 42



One way to make observations is to start with an open mind and brief sets of questions, like those in this slide. This is a distilled version of TRU, suitable for everyday use by a teacher.

In fact, you can have an observation sheet with five columns, with the questions for each dimension at the head of each column.

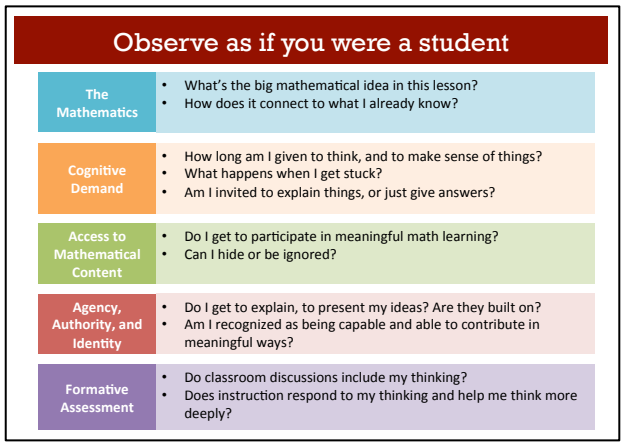
Slide 43



Even better, you can frame the observations from the point of view of the student.

Imagine their experiences, with summaries organized in those five columns.

Slide 44



Of course, TRU also has a research tool that lists classroom activities in order of increasing richness along each dimension. That's in this slide.

What we care about is the developmental trajectory along each dimension – if your classroom looks like it's in the middle along some dimension, what activities might you consider to do better?

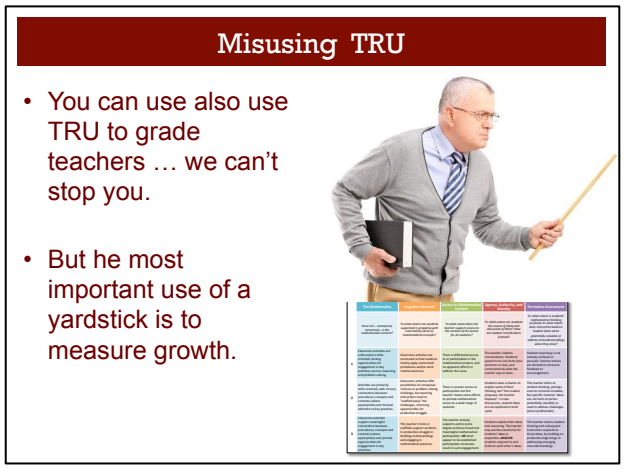
Slide 45

BUT, we recognize that any scoring framework, even if developed for research, can be used to score teachers.

We don't like that – nor is it reasonable.

To be valid and fair, any evaluation needs many dimensions and many samples.

Slide 46



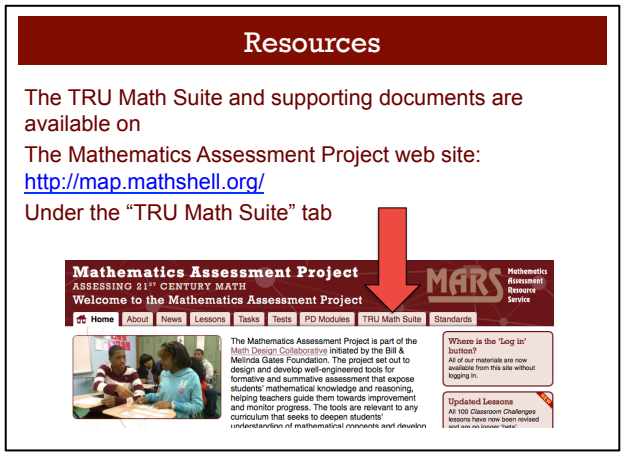
This is where you can find many of our resources.

The big idea is to build professional learning communities that work toward realizing the goals in TRU.

It's worth looking at the MathNIC website for information about other tools that complement this.

Over the next year we hope to develop further tools that can help.

Slide 47



Now's the time for Q&A.

We have time for a brief discussion

At the end, if you are willing

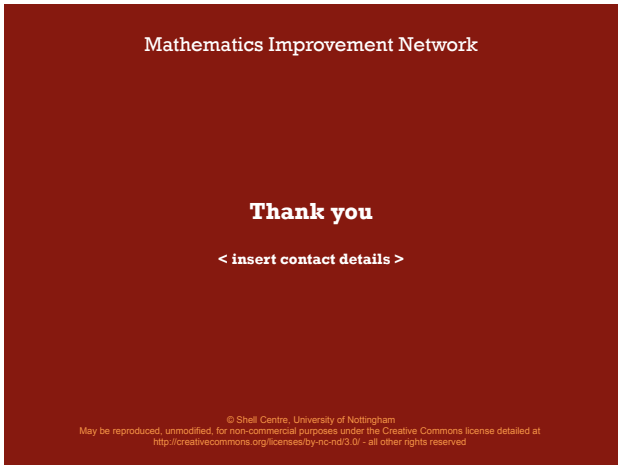
Feel free to contact me later.

Or, perhaps, name another person

Slide 48

Customize this slide with your contact details

Slide 49



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