Mathematics Improvement Network

# Teaching for Robust Understanding

## What makes a mathematically powerful classroom?

# Today's Agenda

1. What really matters in classrooms?
2. Some tools for supporting powerful classroom instruction:
    - Formative Assessment lessons
    - Planning and Reflection
    - Classroom Observation Rubric
3. Q&A, on anything you want to talk about

1. What really matters in classrooms?

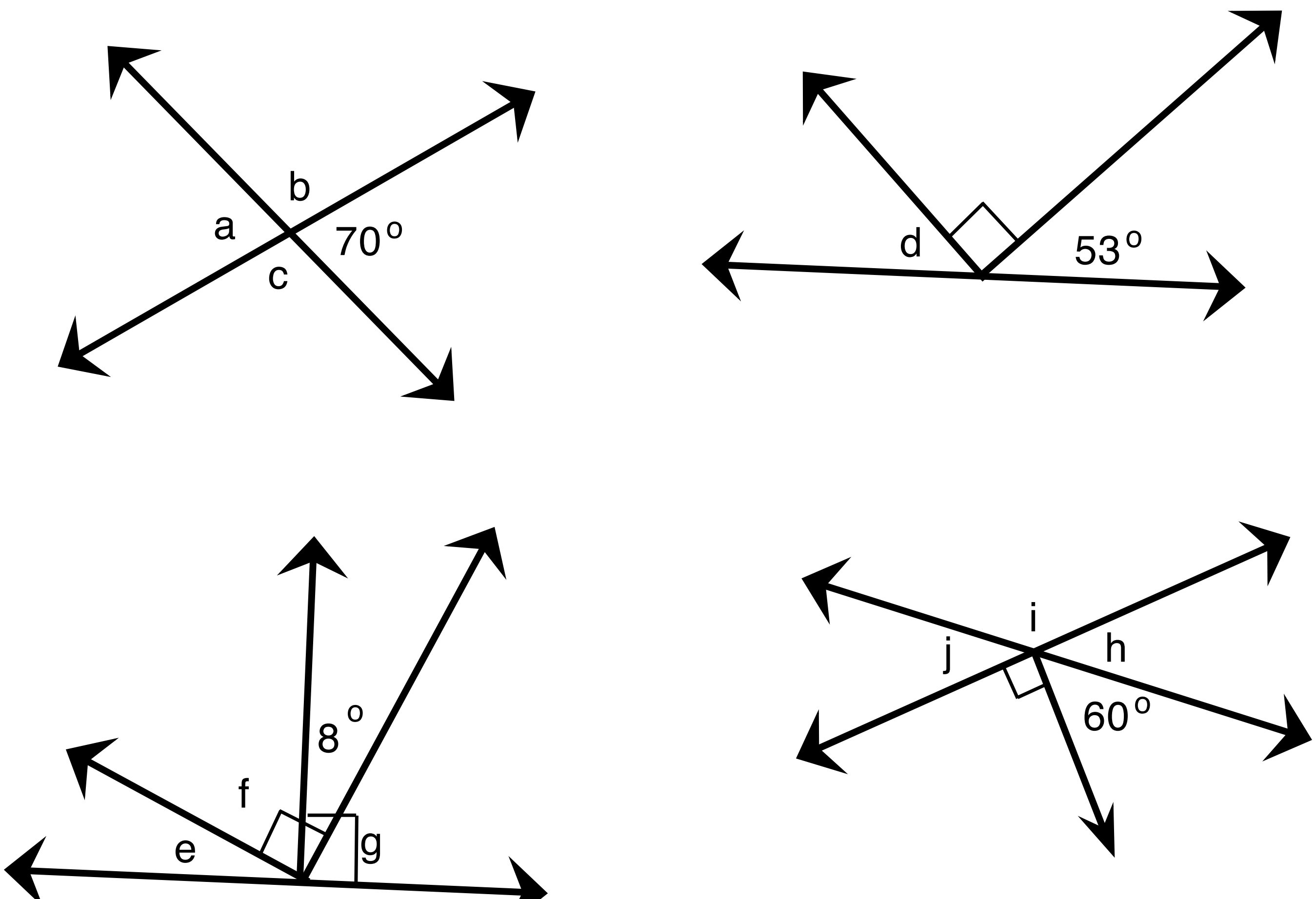
# Establishing a focus

- If you had 5 things to focus on in order to improve mathematics teaching, what would they be?

- How would you know they're the right things?

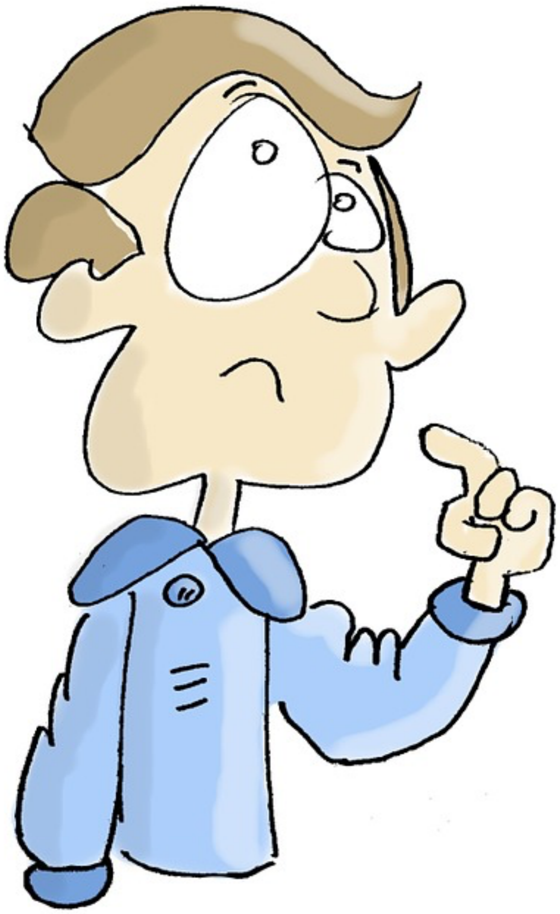
# 1. A lesson on finding angles

Find the measure of all angles



# 1. A lesson on finding angles

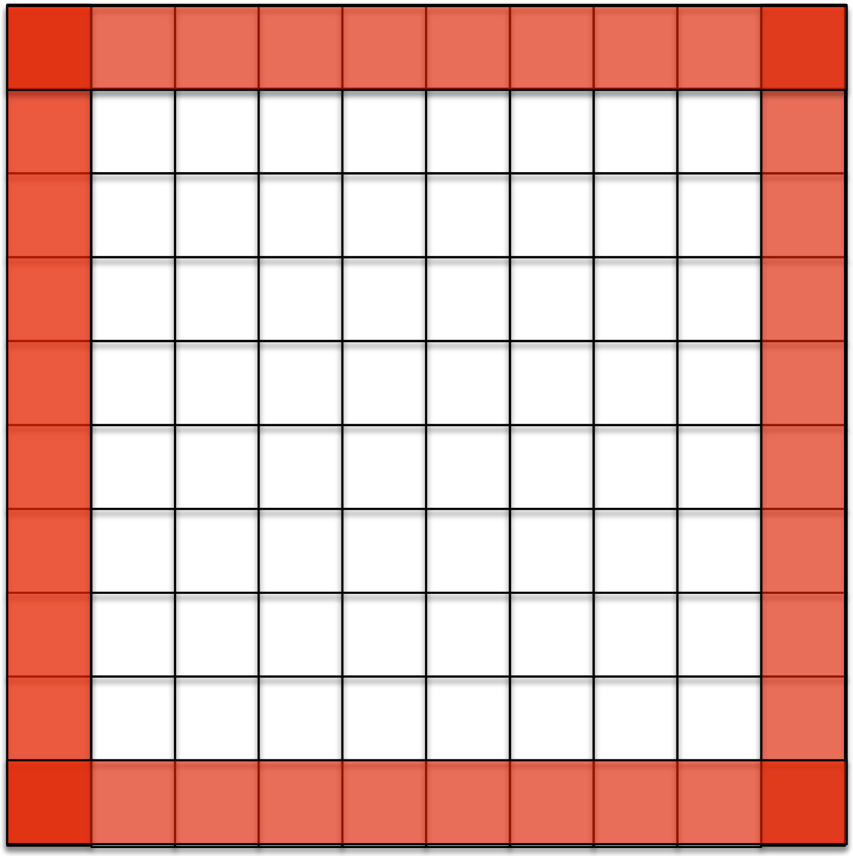
# Think, Pair, Share



- What did you see in the video?
- What was the experience like from the point of view of the students?

# 2. The Border Tiles Lesson

Here’s a 10 x 10 grid.

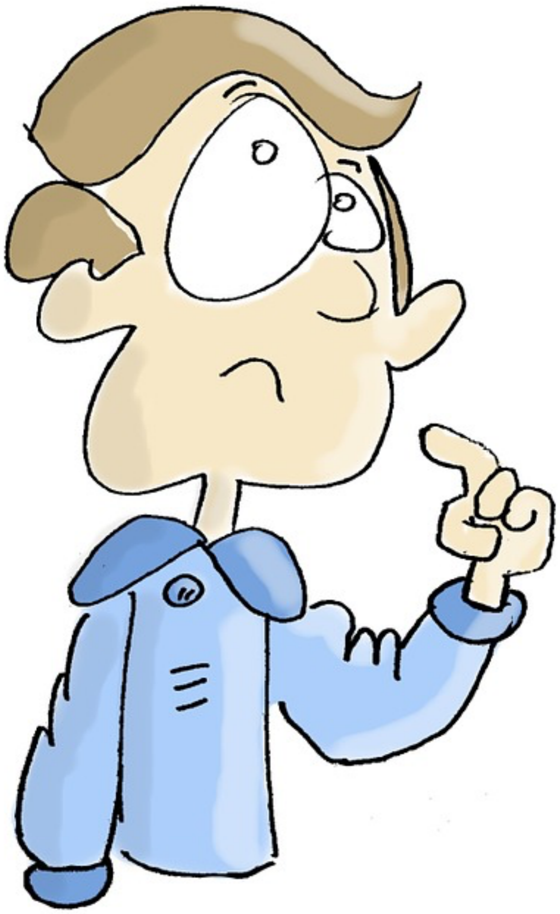


Without counting them all one by one,
how can you figure out the number of red border tiles?

What about other sizes?

# 2. The Border Tiles Lesson

# Think, Pair, Share



- What did you see in the video?
- What was the experience like from the point of view of the students?

# 3. Fractions, decimals percents

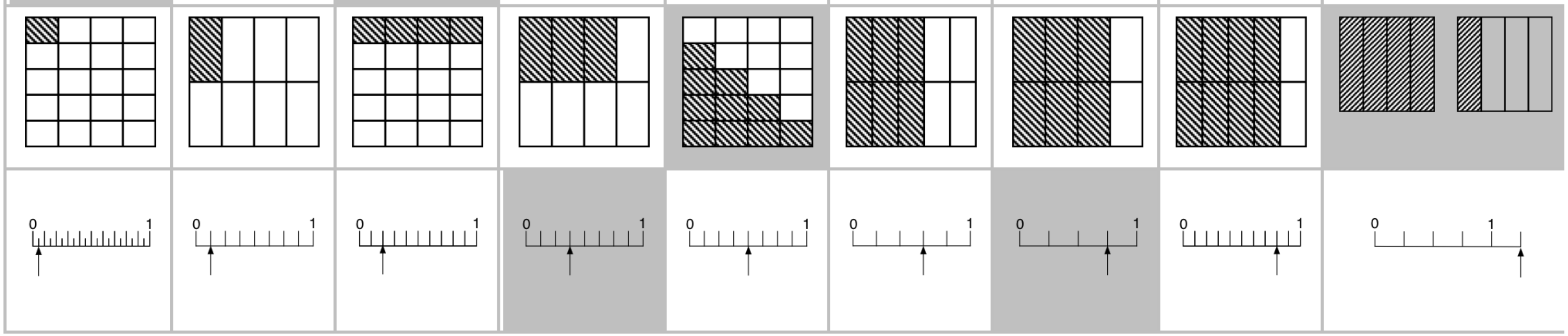
Take turns to:

1. Fill in the missing decimals and percents.
2. Place the cards in order of size.
3. Check that you agree.

| | | |
|---|---|---|
| 0.2 / ___% | 0.05 / ___% | _.___ / 80% |
| 0.375 / ___% | _.___ / 12.5% | 0.75 / ___% |
| 1.25 / ___% | _.___ / 50% | _.___ / ___% |

# 3. Fractions, decimals percents

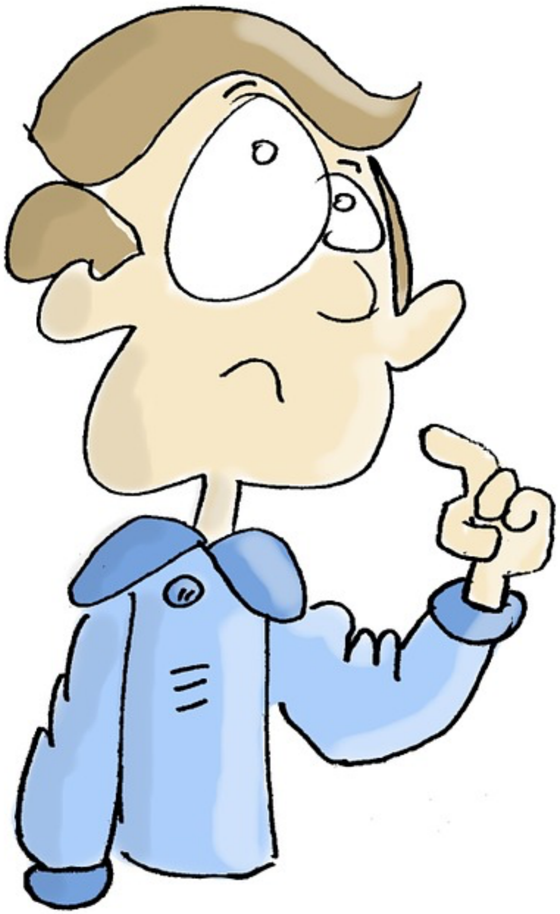
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.05<br>5% | 0.125<br>12.5% | 0.2<br>20% | 0.375<br>37.5% | 0.5<br>50% | 0.6<br>60% | 0.75<br>75% | 0.8<br>80% | 1.25<br>125% |
| $\frac{1}{20}$ | $\frac{1}{8}$ | $\frac{1}{5}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{6}{10}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | $\frac{5}{4}$ |



The gray cards are the ones that students had to create for themselves.

# 3. Fractions, decimals, percents

# Think, Pair, Share



- What did you see in the video?
- What was the experience like from the point of view of the students?

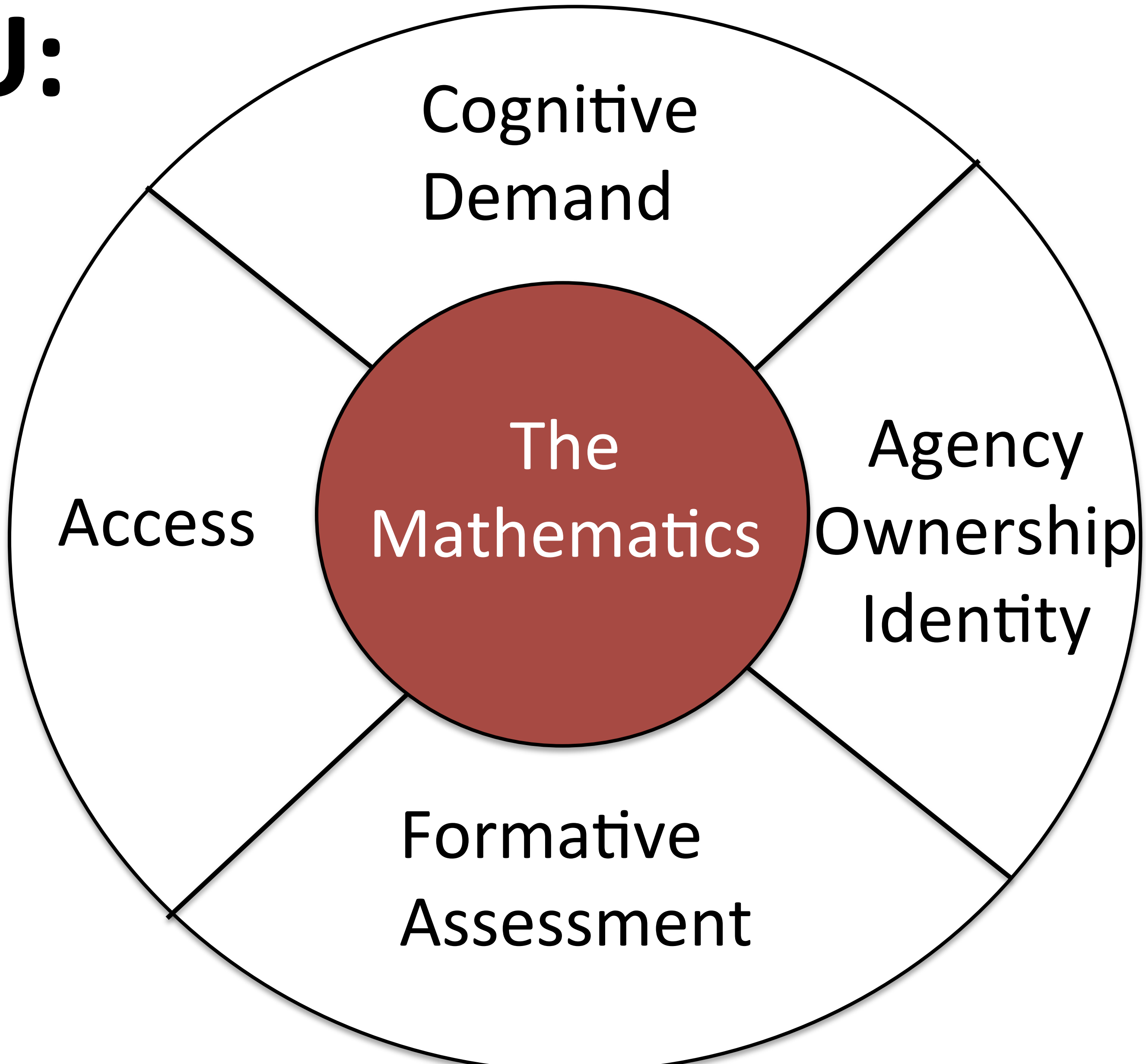
# These were the headings

| | |
|---|---|
| **The Mathematics** | • Is it important, coherent, connected?<br>• Opportunities for thinking and problem solving? |
| **Cognitive Demand** | • Do students have opportunities for sense making?<br>• Do they engage in productive struggle? |
| **Access and Equity** | • Who participates in what ways?<br>• Do *all* students engage in sense-making? |
| **Agency, Ownership** | • Do students have the opportunity to do and talk math?<br>• Do they come to see themselves as math people? |
| **Formative Assessment** | • Does classroom discussion reveal what students understand, so that instruction may be adapted to help students learn? |

# 5 Dimensions of Mathematically Powerful Classrooms

- The Mathematics
- Cognitive Demand
- Access to Mathematical Content
- Agency, Ownership, and Identity
- Formative Assessment

# TRU:


Cognitive Demand
Access
The Mathematics
Agency Ownership Identity
Formative Assessment

# Our claim

Research suggests that:

- Classrooms at all grade levels that do well along these five dimensions will produce students who are powerful mathematical thinkers.
- Instructional materials, professional development, and classroom observations will be most powerful if they are aligned with these five dimensions.

# 2. Tools for supporting powerful classroom instruction

a. Formative Assessment lessons

b. Planning and Reflection

c. Ways to observe classrooms

# Tool (a) Formative Assessment Lessons

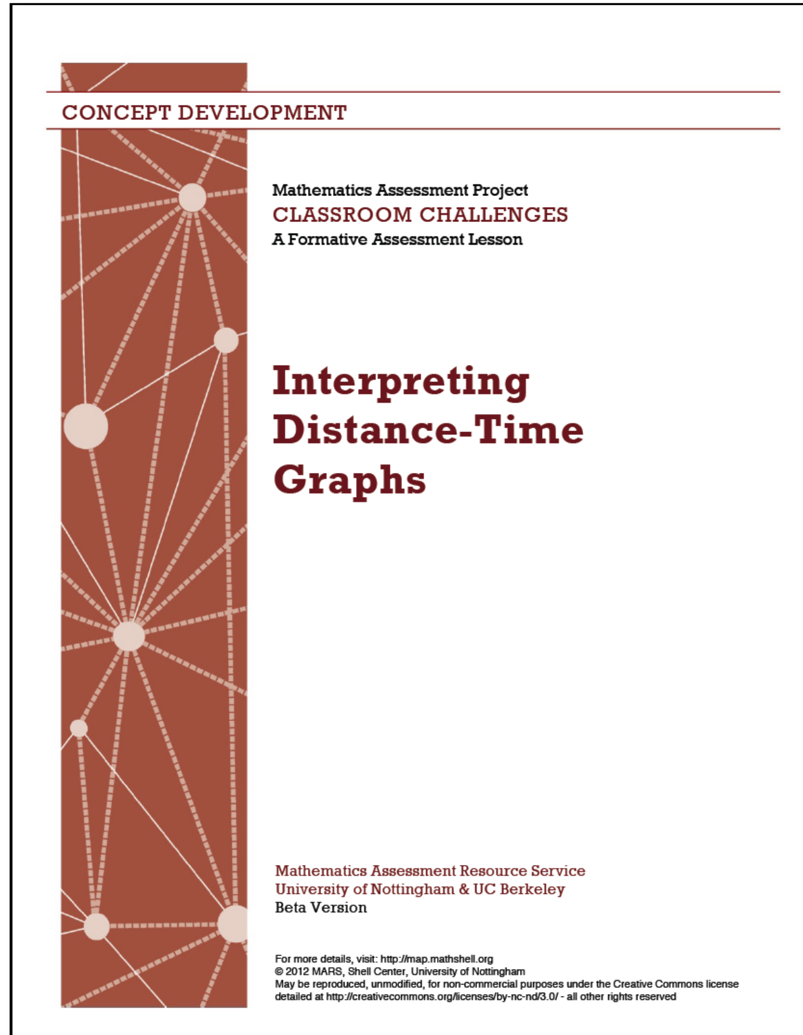
http://map.mathshell.org

100 Classroom Challenges:

- 20 at each grade 6-8
- 40 for Career and College readiness at High School.

- 70% concept focused
- 30% problem solving

Each lesson attempts to integrate formative assessment into everyday teaching.


CONCEPT DEVELOPMENT

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

**Interpreting Distance-Time Graphs**

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

For more details, visit: http://map.mathshell.org
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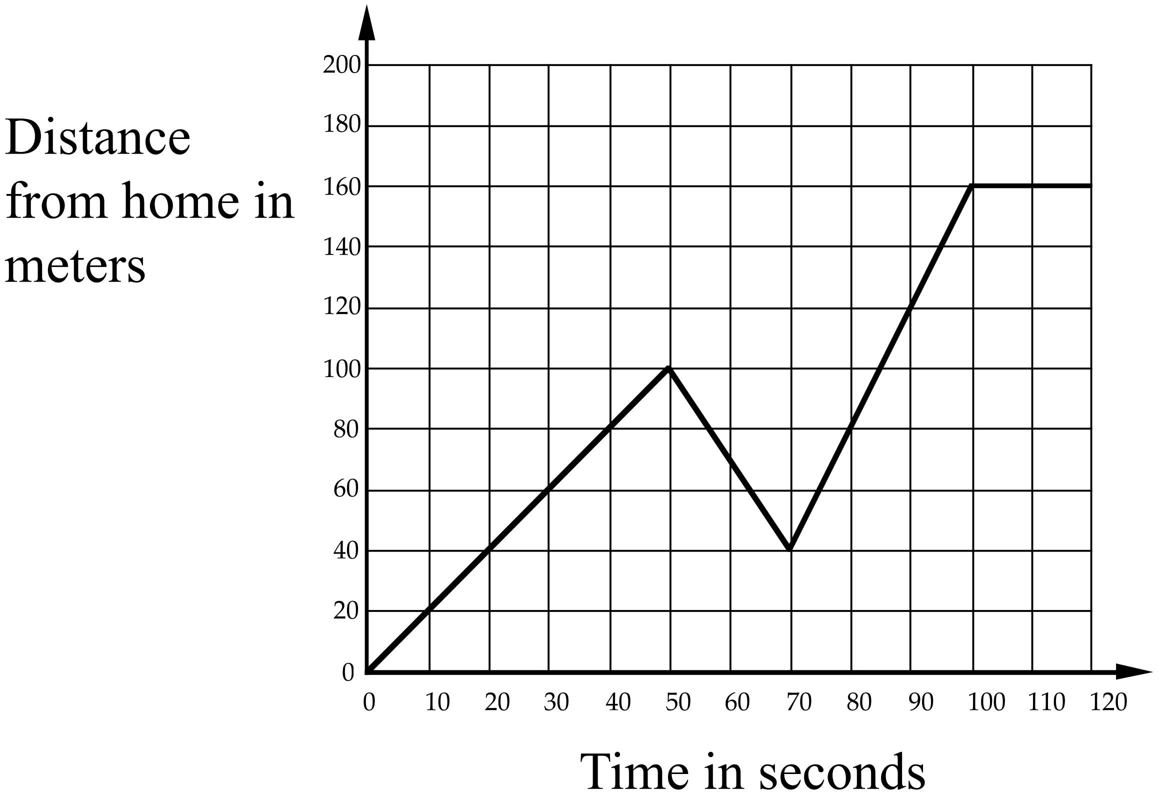
# Diagnostic task before the lesson

Every morning Jane walks along a straight road to a bus stop 160 meters from her home, where she catches a bus to college.

The graph shows her journey on one particular day.

Describe what may have happened.

Is the graph realistic? Why?



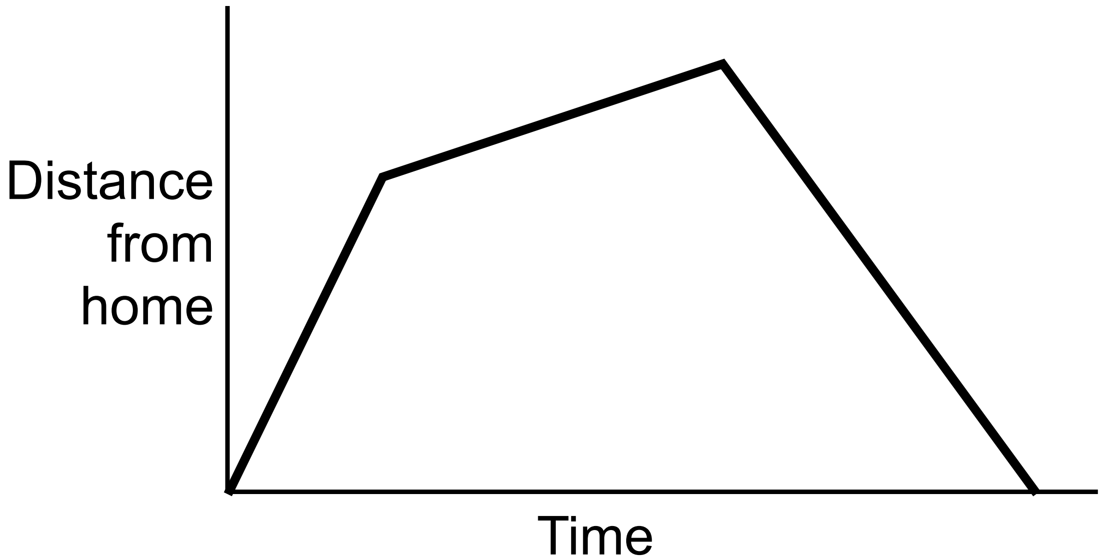
| Common Issue | Possible questions and prompts |
|---|---|
| **Student interprets the graph as a picture**<br>E.g. as the graph goes up and down, Tom's path goes up and down. | • If a person walked at a steady speed up and down a hill, *directly away from home*, what would the graph look like? |
| **Student interprets graph as speed–time**<br>E.g. The student has interpreted a positive slope as speeding up and a negative slope as slowing down. | • How can you tell if Tom is traveling away from or towards home? |
| **Student fails to mention distance or time**<br>E.g. The student has not worked out the speed of some/all sections of the journey. | • Can you provide more information about how far Tom has traveled during different sections of his journey? |
| **Student fails to calculate and represent speed** | • Can you provide information about Tom's speed for all sections of his journey? |
| **Student adds little explanation as to why the graph is or is not realistic** | • Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why?/Why not? |

# Lesson beginning: Which story best fits?

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

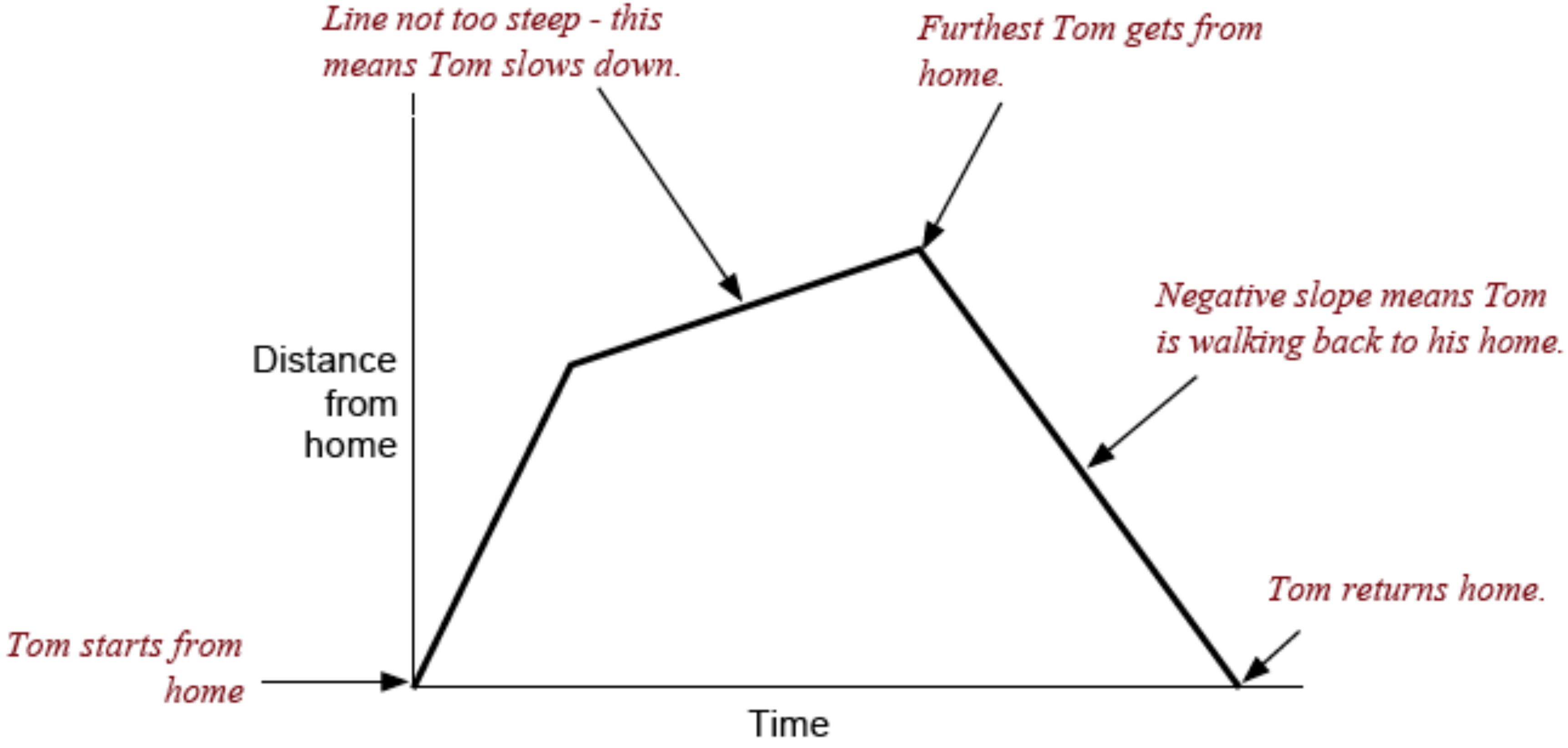
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.

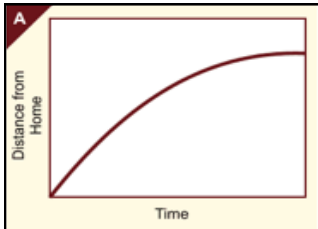


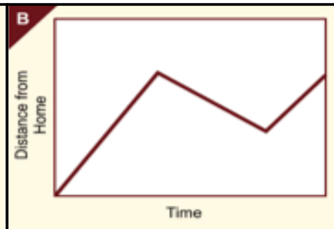
# Students are given the chance to annotate and explain…

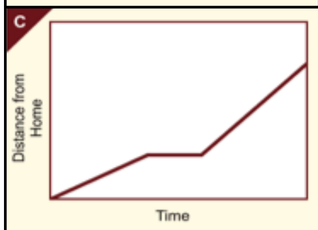
A graph may end up looking like this:

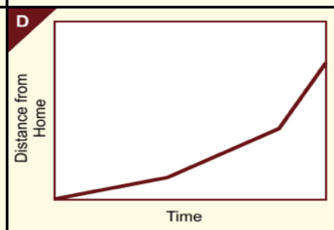


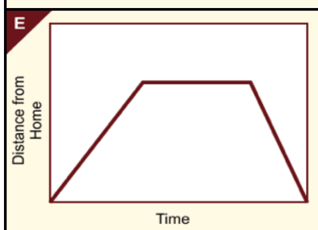
# Matching graphs and stories

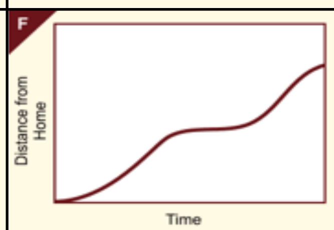


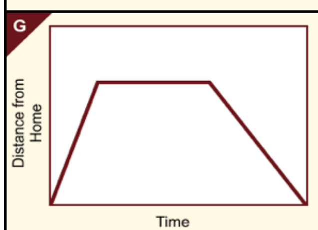



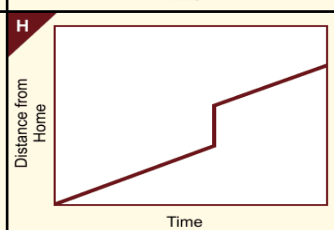



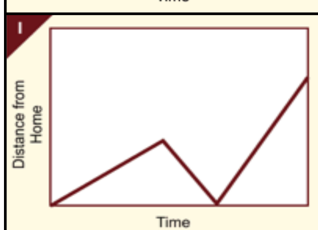



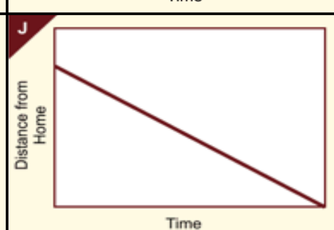



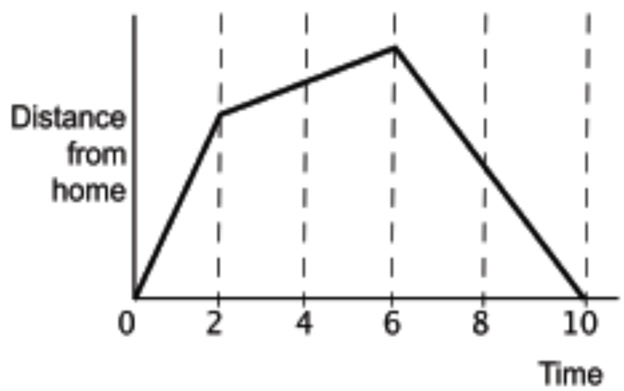

1. Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.
2. Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
3. Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.
4. Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
5. Tom left his home for a run, but he was unfit and gradually came to a stop!
6. Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7. Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.
8. This graph is just plain wrong. How can Tom be in two places at once?
9. After the party, Tom walked slowly all the way home.
10. Make up your own story!

# Students work on converting graphs to tables:

**Whole-class discussion: Interpreting tables (15 minutes)**

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide 5 of the projector resource:

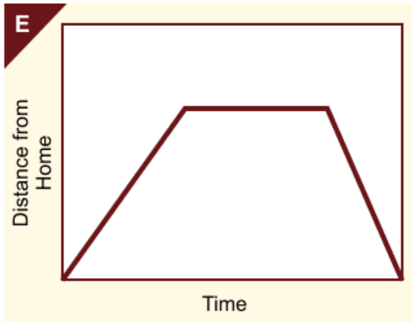
## Making Up Data for a Graph

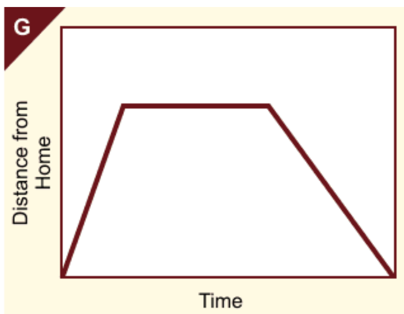


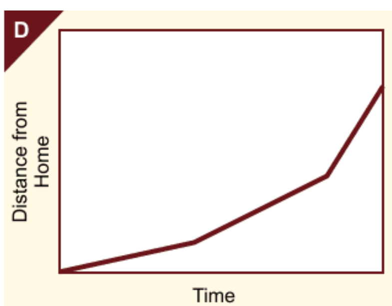

| Time | Distance |
|---|---|
| 0 | |
| 2 | |
| 4 | |
| 6 | |
| 8 | |
| 10 | |

*On your whiteboard, create a table that shows possible times and distances for Tom's journey.*

# Tables are added to the card sort…




**2**

Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

**1**

Tom ran from his home to the bus stop and waited.
He realized that he had missed the bus so he walked home.

**6**

Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

**Q**

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 40 |
| 4 | 60 |
| 5 | 120 |

**P**

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 40 |
| 2 | 40 |
| 3 | 40 |
| 4 | 20 |
| 5 | 0 |

**T**

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 20 |
| 2 | 40 |
| 3 | 40 |
| 4 | 40 |
| 5 | 0 |

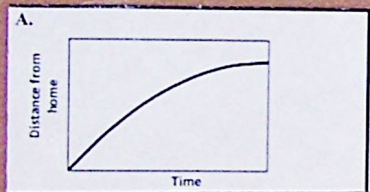
And the class compares solutions together

5
Tom left his home for a run, but he was unfit and gradually came to a stop!

B.

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 40 |
| 4 | 60 |
| 5 | 120 |

A.

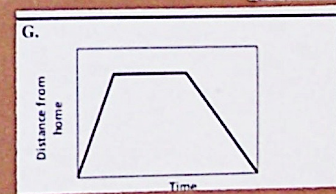



6.
Tom walked to the store at the end of his street, bought a newspaper, then ran all the way back.

A.

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 40 |
| 2 | 40 |
| 3 | 40 |
| 4 | 20 |
| 5 | 0 |

G.

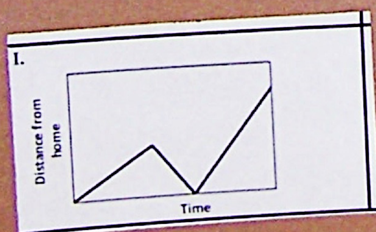



7.
Tom went out for a walk with some friends when he suddenly realised he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.

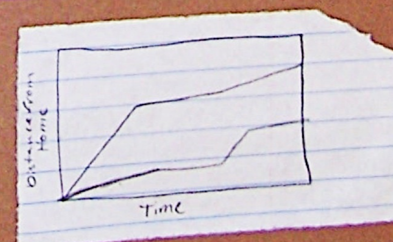
F.

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 30 |
| 2 | 60 |
| 3 | 0 |
| 4 | 60 |
| 5 | 120 |

I.




8.
This graph is just plain wrong. How can Tom be in two places at once?




K.

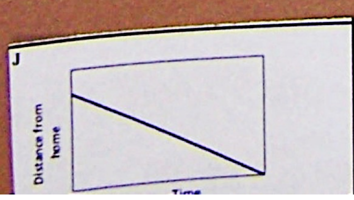
| Time | Dist. |
|---|---|
| 1 | 0 |
| 2 | 20 |
| 3 | 40 |
| 4 | 60 |
| 5 | 80 |
| | 0 |
| 1 | 10 |
| 2 | 10 |
| 3 | 14 |
| 4 | 20 |
| 5 | 20 |

9.
After the party, Tom walked slowly all the way home.

I

| Time | Distance |
|---|---|
| 0 | 120 |
| 1 | 96 |
| 2 | 72 |
| 3 | 48 |
| 4 | 24 |
| 5 | 0 |

J

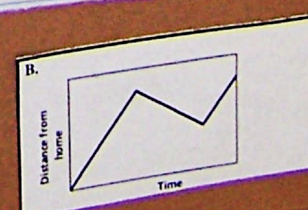



10.
Tom jogged to the park wanted a water jogged back to the store then jogged back to the park.

D.

| Time | Distance |
|---|---|
| 0 | 0 |
| 1 | 40 |
| 2 | 80 |
| 3 | 60 |
| 4 | 40 |
| 5 | 80 |

B.

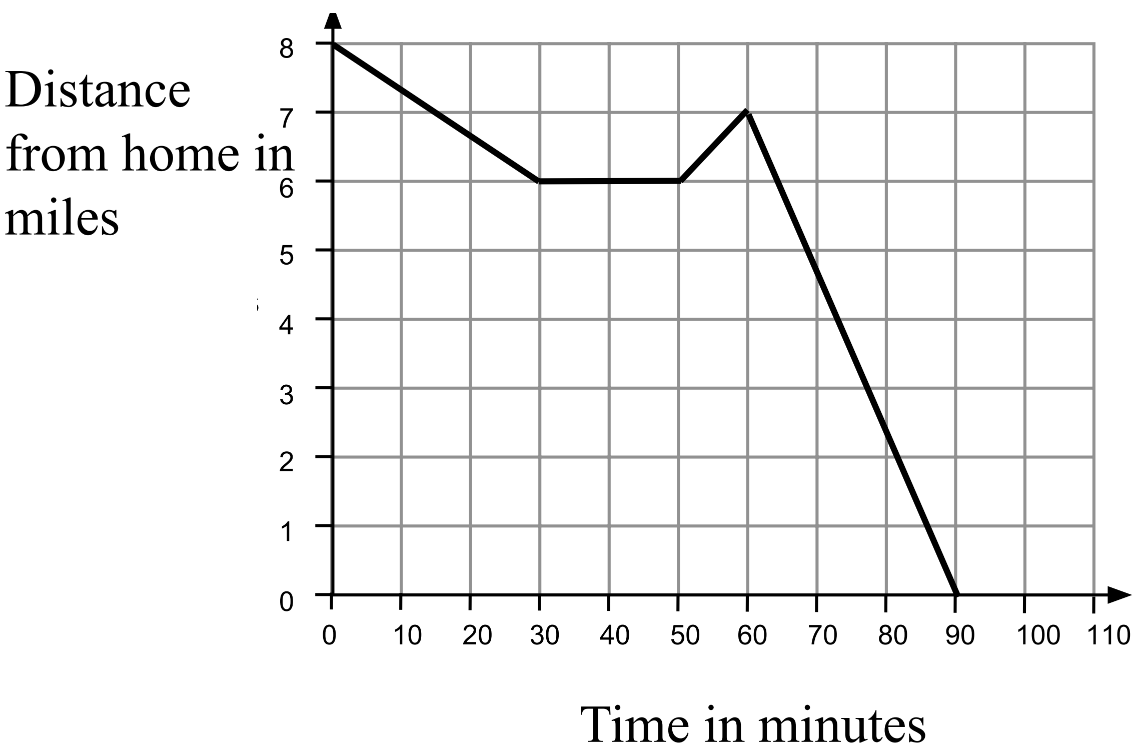


# End of lesson task

Sylvia bikes along a straight road from her friend's house home, a distance of 7 miles. The graph shows her journey.

Describe what may have happened.

Include details like how fast she bikes.



## The Mathematics

*How rich – connected, conceptual – is the mathematical content?*

The lesson focuses on developing deep understandings of concepts like slope, and its use to describe real world phenomena; it provides opportunities to make connections across different representations (graphs, tables, stories).

# Cognitive Demand

*To what extent are students supported in grappling with and making sense of mathematical concepts?*

The card sort and poster activities provide plenty of room for sense making – *if* the students are gently supported when they need it. (Remember the list of support questions)

## Access to Mathematical Content

*To what extent does the teacher support access to the content of the lesson for all students?*

The classroom structures – which include whole group conversations, small group work, and student poster presentations – provide *opportunities* for teachers to support every student in engaging meaningfully with the mathematics. But . . . this takes hard work, even with the opportunities.

## Agency, Ownership, and Identity

*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*

The classroom structures – which include whole group conversations, small group work, and student poster presentations – provide *opportunities* for teachers to support every student in building powerful mathematical identities. But . . . this takes hard work, even with the opportunities.

## Formative Assessment

*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*

They're known as

**Formative Assessment Lessons**

for a reason…

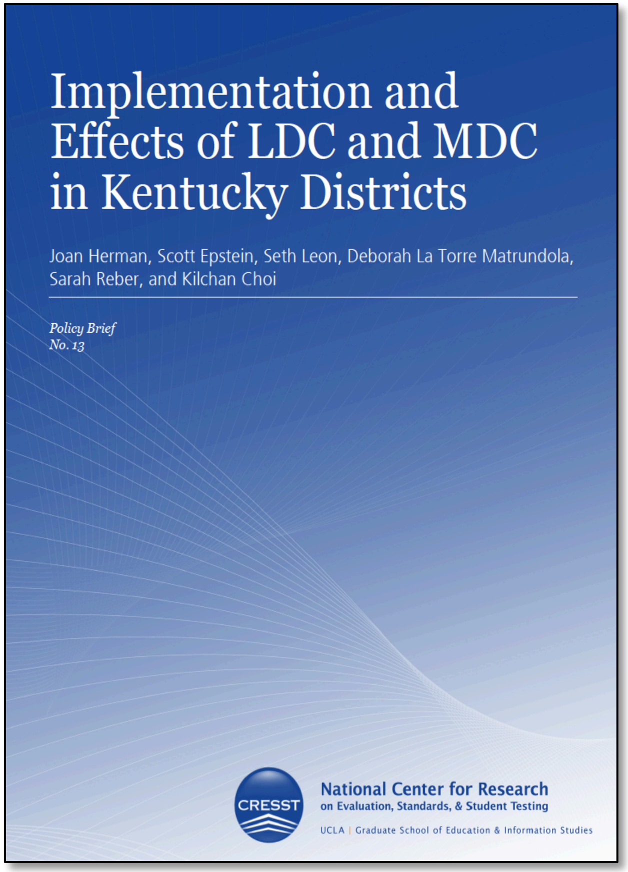
☺

# Is this type of lesson effective?

The National Center for Research on Evaluation, Standards and Student Testing (CRESST)

assessed the effectiveness of the Math Design Collaborative's (MDC) use of Classoom Challenges in 9th Grade Kentucky classrooms.

What did they find?



# Is this type of lesson effective?

- "For MDC, participating teachers were expected to implement between four and six Challenges, meaning that **students were engaged only 8-12 days of the school year**

- Nonetheless, the studies found **statistically significant learning effects… the approximate equivalent of 4.6 months for MDC**. Given their contexts of early implementation and limited dosage, these small effects are noteworthy."

# Tool (b) Planning and Reflection

- The TRU Math Conversation Guide.

- The TRU dimensions become arenas for teachers to reflect on their own teaching:

  - in planning,
  - in reflecting on how things have gone
  - in thinking about next steps.

## TRU Math Conversation Guide:

### A Tool for Teacher Learning and Growth[1]

This *TRU Math Conversation Guide* is a product of The Algebra Teaching Study (NSF Grant DRL-0909815 to PI Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL-0909851 to PI Robert Floden, Michigan State University), and of The Mathematics Assessment Project (Bill and Melinda Gates Foundation Grant OPP53342 to PIs Alan Schoenfeld, U. C Berkeley, and Hugh Burkhardt and Malcolm Swan, The University of Nottingham).

A companion document, the *TRU Math Conversation Guide, Module A: Contextual Algebraic Tasks,* supports in-depth explorations of algebraic thinking, with a focus on complex modeling and applications problems. *Module A: Contextual Algebraic Tasks* is the first of a series of content-specific conversation guides aimed at supporting classroom engagement with centrally important mathematical ideas. The *TRU Math Conversation Guide Modules* will all be accessible at http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/index.php.

Suggested citation:

Baldinger, E., & Louie, N. *TRU Math conversation guide: A tool for teacher learning and growth*. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from: http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/pd.php.



[1] You are reading the first public version of this conversation guide. We hope that reflecting on teaching in the ways suggested here will be productive. We also welcome comments and suggestions for improvement. Please contact Nicole (NLL@berkeley.edu) and Evra (evra@berkeley.edu) with your feedback.

1

# Start with the core questions:

## The Mathematics

*How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?*

## Cognitive Demand

*What opportunities do students have to make their own sense of mathematical ideas?*

## Access to Mathematical Content

*Who does and does not participate in the mathematical work of the class, and how?*

## Agency, Ownership, and Identity

*What opportunities do students have to explain their own and respond to each other's mathematical ideas?*

## Formative Assessment

*What do we know about each student's current mathematical thinking, and how can we build on it?*

# . . . and expand them.

- **Before a lesson**, you can ask:
  - How can I use the five dimensions to enhance my lesson planning?
- **After a lesson**, you can ask:
  - How well did things go? What can I do better next time?
- **Planning next Steps**, you can ask:
  - How can I build on what I've learned?

# Looking at the conversation guide

## TRU Math Conversation Guide:

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## Access to Mathematical Content

*Core Question: Who does and does not participate in the mathematical work of the class, and how*

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

## Access to Mathematical Content

| Pre-observation | Reflecting After a Lesson | Planning Next Steps |
|---|---|---|
| What opportunities exist for each student to participate in the mathematical work of the class? | Who did and didn't participate in the mathematical work of the class, and how? | How can we create opportunities for each student to participate in the mathematical work of the class? |

*Think about:*

- The range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions.
- Language demands and the development of students' academic language.
- How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students.
- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).

## The Mathematics

*Core Question: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?*

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to

## Cognitive Demand

*Core Question: What opportunities do students have to make their own sense of mathematical ideas*

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in

## Agency, Authority, and Identity

*Core Question: What opportunities do students have to explain their own and respond to each other's mathematical ideas?*

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize.

## Formative Assessment

*Core Question: What do we know about each student's current mathematical thinking, and how can we build on it?*

We want instruction to be responsive to students' actual thinking, not just our hopes or assumptions about what they do and don't understand. It isn't always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students' understandings, partial though they may be, and build on them.

## Formative Assessment

| Pre-observation | Reflecting After a Lesson | Planning Next Steps |
|---|---|---|
| What do we know about each student's current mathematical thinking, and how does this lesson build on it? | What did we learn in this lesson about each student's mathematical thinking? How was this thinking built on? | Based on what we learned about each student's mathematical thinking, how can we (1) learn more about it and (2) build on it? |

*Think about:*

- What opportunities exist for students to develop their own strategies and approaches.
- What opportunities exist for students to share their mathematical ideas and reasoning, and to connect their ideas to others'.
- What different ways students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- Who students get to share their ideas with (e.g., a partner, the whole class, the teacher).
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- What we know and don't know about how each student is making sense of the mathematics we are focusing on.
- What opportunities exist to build on students' mathematical thinking, and how teachers and/or other students take up these opportunities.

# Access to Mathematical Content

*Core Question: Who does and does not participate in the mathematical work of the class, and how*

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

# Access to Mathematical Content

| Pre-observation | Reflecting After a Lesson | Planning Next Steps |
|---|---|---|
| What opportunities exist for each student to participate in the mathematical work of the class? | Who did and didn't participate in the mathematical work of the class, and how? | How can we create opportunities for each student to participate in the mathematical work of the class? |

*Think about:*

- The range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
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- Language demands and the development of students' academic language.
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- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).

# Tool (c) Ways to Observe Classrooms

Here are three ways of observing:

- Observe as a teacher.
- Observe, as a student.
- Observe as a researcher.

# Observe as a teacher

| | |
|---|---|
| **The Mathematics** | • Are students learning important mathematics?<br>• Are opportunities made for meaningful connections? |
| **Cognitive Demand** | • How long do students spend on each prompt?<br>• Do they engage in productive struggle?<br>• Do teacher questions invite explanations or answers? |
| **Access to Mathematical Content** | • Are there multiple ways to get involved productively?<br>• Does the teacher ask a range of students to respond? |
| **Agency, Authority, and Identity** | • Who explains most: the teacher or the students?<br>• Do the students give extended explanations? |
| **Formative Assessment** | • Does the teacher follow up on student responses?<br>• Does the teacher vary the lesson in the light of student responses? |

# Observe as if you were a student

| | |
|---|---|
| **The Mathematics** | • What's the big mathematical idea in this lesson?<br>• How does it connect to what I already know? |
| **Cognitive Demand** | • How long am I given to think, and to make sense of things?<br>• What happens when I get stuck?<br>• Am I invited to explain things, or just give answers? |
| **Access to Mathematical Content** | • Do I get to participate in meaningful math learning?<br>• Can I hide or be ignored? |
| **Agency, Authority, and Identity** | • Do I get to explain, to present my ideas? Are they built on?<br>• Am I recognized as being capable and able to contribute in meaningful ways? |
| **Formative Assessment** | • Do classroom discussions include my thinking?<br>• Does instruction respond to my thinking and help me think more deeply? |

# Observe as if you were a researcher

| | The Mathematics | Cognitive Demand | Access to Mathematical Content | Agency, Authority, and Identity | Formative Assessment |
|---|---|---|---|---|---|
| | *How rich – conceptual, connected – is the mathematical content?* | *To what extent are students supported in grappling with and making sense of mathematical concepts?* | *To what extent does the teacher support access to the content of the lesson for all students?* | *To what extent are students the source of ideas and discussion of them? How are student contributions framed?* | *To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?* |
| **1** | Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving. | Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises. | There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue. | The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does. | Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement. |
| **2** | Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices. | Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle. | There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students. | Students have a chance to explain some of their thinking, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon. | The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic). |
| **3** | Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices. | The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices. | The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement. | Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas. | The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. |

# Misuses of TRU

- You can use also use TRU to grade teachers … we can't stop you.
- But the most important use of a yardstick is to measure growth.

| | The Mathematics | Cognitive Demand | Access to Mathematical Content | Agency, Authority, and Identity | Formative Assessment |
|---|---|---|---|---|---|
| | *How rich – conceptual, connected – is the mathematical content?* | *To what extent are students supported in grappling with and making sense of mathematical concepts?* | *To what extent does the teacher support access to the content of the lesson for all students?* | *To what extent are students the source of ideas and discussion of them? How are student contributions framed?* | *To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?* |
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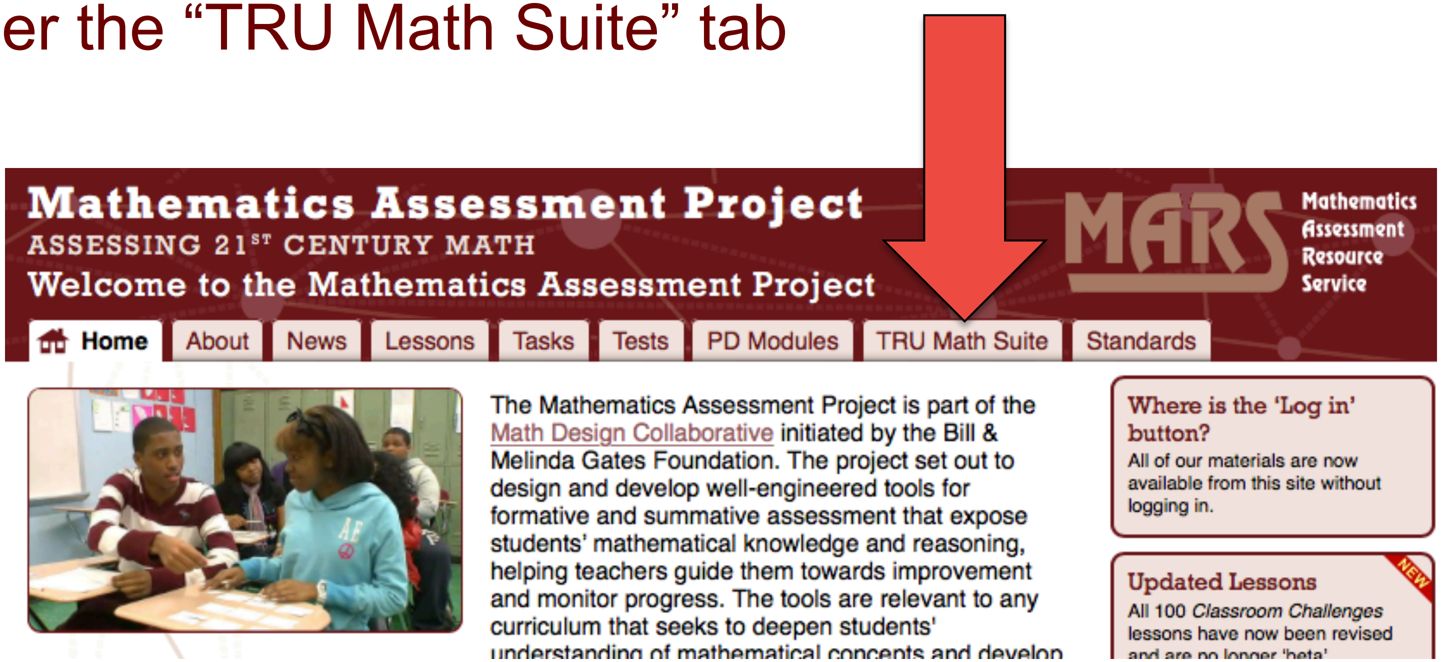
# Resources

The TRU Math Suite and supporting documents are available on

The Mathematics Assessment Project web site: http://map.mathshell.org/

Under the “TRU Math Suite” tab



# Q&A

Questions?

Comments?

Suggestions?

# Thank you

**< insert contact details >**