Mathematical Practices

Handouts

Contents

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Handout 1: Common Core State Standards for Mathematical Practice

MP1: Make sense of problems and persevere in solving them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2: Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3: Construct viable arguments and critique the reasoning of others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP4: Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5: Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6: Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7: Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$,older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8: Look for and express regularity in repeated reasoning

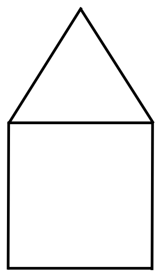
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Source: http://www.corestandards.org/Math/Practice/

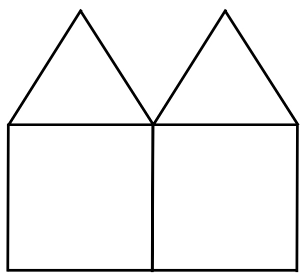
Handout 2: Matchstick Houses

Joseph uses matchsticks to make rows of houses:

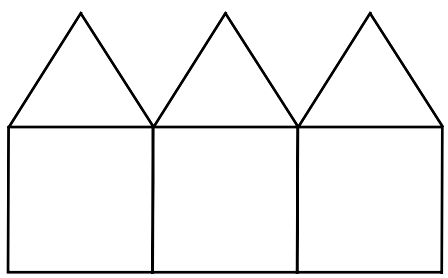
1 house



2 houses



3 houses



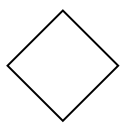
1. How many matchsticks are needed to make 6 houses in a row?

2. Find a rule for the number of matchsticks needed to make a given number of houses.

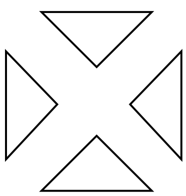
Handout 3: Table Tiling Task

Maria makes square tables, then sticks tiles to the top.

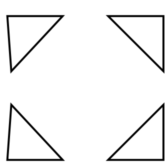
She uses three types of tiles:



whole tiles



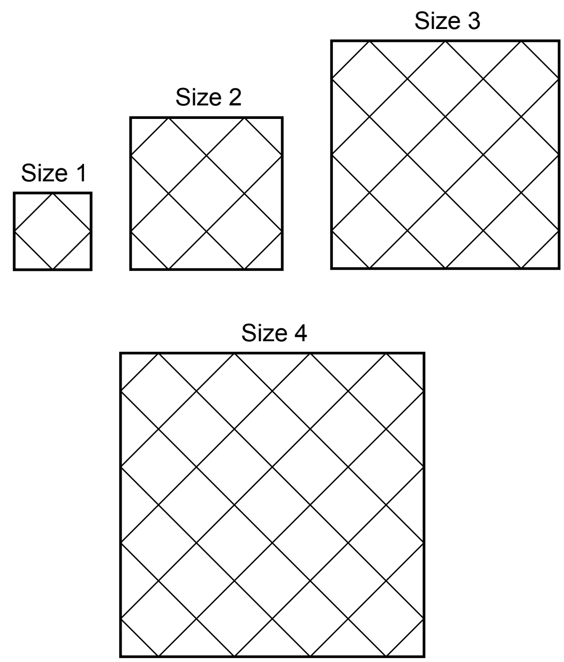
half tiles



quarter tiles

Maria only uses quarter tiles in the corners and half tiles along the edges of the table.

Here are four table tops:



1. Complete this table to show how many whole tiles, half tiles, and quarter tiles she needs for each of these sizes.

Size (n)	Number of whole tiles (w)	Number of half tiles (h)	Number of quarter tiles (q)
1	1	0	4
2	5	4	
3	13	8	
4			

2. Find a rule, or a formula, that will help Maria figure out the number of half tiles that she needs for tables of different sizes. Explain how your rule works.

3. Use the number patterns in the table to find a rule, or a formula, that will help Maria figure out the number of whole tiles Maria needs for tables of different sizes. Explain why your rule works.

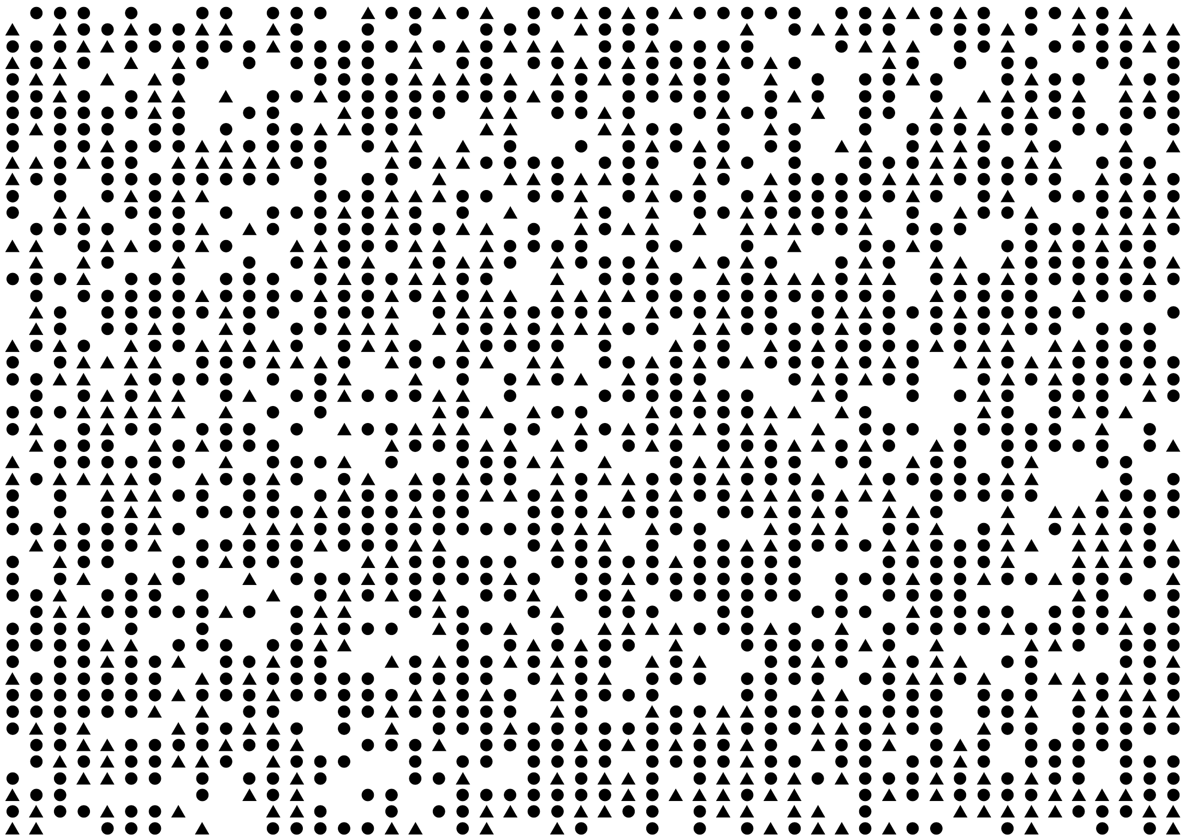
4. Maria has made a table with 20 half tiles. How many whole tiles are on this table? Show how you found the number of whole tiles.

Handout 4: Identifying Mathematical Practices

Write down the practice(s) students have opportunity to develop when completing the Table Tiling task, identifying at what point in the solution process different practices emerge:

MP1:
MP2:
MP3:
MP4:
MP5:
MP6:
MP7:
MP8:

Handout 5: Sampling and Estimating - Counting Trees



This diagram shows some trees in a tree farm.

The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type?
   Explain your method fully.

2. Use your method to estimate the number of:
   (a) Old trees
   (b) Young trees

Source: http://map.mathshell.org/lessons.php?unit=7400&collection=8

Handout 6: Interpreting the Mathematical Practices

Mathematical Practice	Student-friendly language
Make sense & persevere in solving problems	I don't mind trying many times to understand and solve a math problem.
Reason abstractly & quantitatively	I can think through a problem step by step, using numbers, graphs and math symbols, checking back with the problem each time.
Construct viable arguments & critique the reasoning of others	I can explain how I solved the problem and why my reasoning is correct. I can discuss other students' solutions too.
Model with mathematics	When faced with a real-world problem, I can make sense of it, then use pictures, numbers, tables, graphs, and symbols to represent it and work out a solution.
Use appropriate tools strategically	I can choose the right math tool for solving a problem: like calculators, rules, pictures, or objects.
Attend to precision	I can make sure my method and my calculations are correct and my explanation is complete and convincing, using clear mathematical language.
Look for and make use of structure	I can use what I already know about math to solve the problem, using equivalent expressions to make sense of things.
Look for and express regularity in repeated reasoning	I can look for a pattern in a problem, whenever the reasoning repeats. I can represent the pattern by a rule or formula.

Adapted from: Teaching Children Mathematics, March 2012*, Getting Started in K – Grade 2*, pp. 440-445

Handout 7: Common Core State Standards Math Practice Charts

Make sense of problems and persevere in solving them

When presented with a problem, I can make a plan, carry out my plan, and evaluate its success.

BEFORE...

EXPLAIN the problem to myself.

- *Have I solved a problem like this before?*

ORGANIZE information...

- *What is the question I need to answer?*
- *What is given?*
- *What is not given?*
- *What are the relationships between known and unknown quantities?*
- *What tools will I use?*
- *What prior knowledge do I have to help me?*

DURING...

PERSEVERE

MONITOR my work

CHANGE my plan if it isn't working out

ASK myself, "Does this make sense?"

AFTER...

CHECK

- *Is my answer correct?*
- *How do my representations connect to my algorithms?*

EVALUATE

- *What worked?*
- *What didn't work?*
- *What other strategies were used?*
- *How was my solution similar to or different from my classmates'?*

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Reason abstractly and quantitatively

I can use reasoning habits to help me contextualize and decontexualize problems.

CONTEXTUALIZE

I can take numbers and put them in a real-world context.

For example, if given
3 x 2.5 = 7.5
I can create a context:

I walked 2.5 miles per day for 3 days. I walked a total of 7.5 miles.

DECONTEXTUALIZE

I can take numbers out of context and work mathematically with them.

For example, if given
'I walked 2.5 miles per day for 3 days. How far did I walk?',
I can write and solve

3 x 2.5 = 7.5

Reasoning Habits include *1) creating an understandable representation of the problem solved, 2) considering the units involved, 3) attending to the meaning of quantities, and 4) using properties to help solve problems.*

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Source: http://www.debbiewaggoner.com/uploads/1/2/9/9/12998469/math_practices_posters_ms_hs.pdf

Construct viable arguments and critique the reasoning of others

I can make conjectures and critique the mathematical thinking of others.

I can ***construct***, ***justify***, and ***communicate*** arguments by…

- considering context
- using examples and non-examples
- using objects, drawings, diagrams and actions

I can ***critique the reasoning of others*** by…

- listening
- comparing arguments
- identifying flawed logic
- asking questions to *clarify* or *improve arguments*

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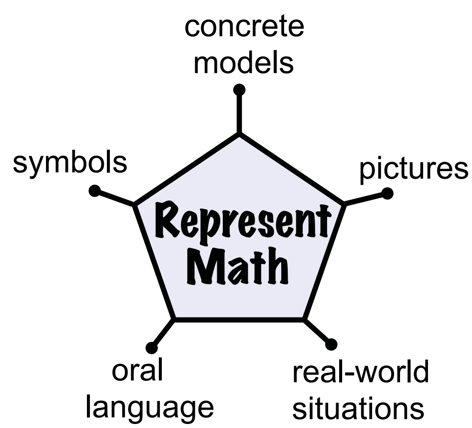
Model with mathematics

I can recognize math in everyday life and use math I know to solve everyday problems.

I can…

- make assumptions and estimate to make complex problems easier
- identify important quantities and use tools to show their relationships
- evaluate my answer and make changes if needed



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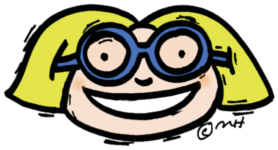
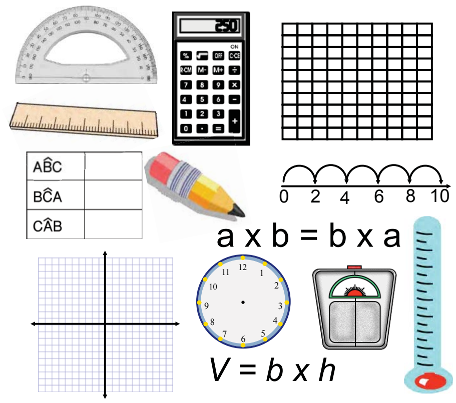
Use appropriate tools strategically



I know when to use certain tools to help me explore and deepen my math understanding.

I have a math toolbox.

- I know *HOW* to use math tools.
- I know *WHEN* to use math tools.
- I can reason: *"Did the tool I used give me an answer that makes sense?"*



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Attend to precision

I can use precision when solving problems and communicating my ideas.

Problem Solving

- I can calculate *accurately*.
- I can calculate *efficiently*.
- My answer matches what the problem asked me to do – *estimate* or find an *exact answer*.

Communicating

- I can SPEAK, READ, WRITE, and LISTEN mathematically.
- I can correctly use…
  - math symbols
  - math vocabulary
  - units of measure

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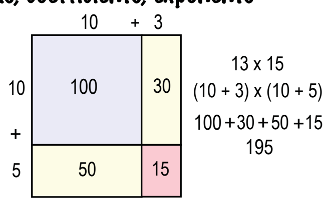
Look for and make use of structure

I can see and understand how numbers and spaces are organized and put together as parts and wholes.

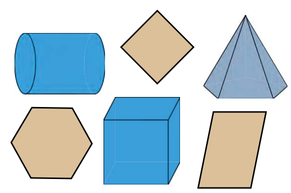
Numbers

For Example:
- Base 10 structure
- operations and properties
- terms, coefficients, exponents



Spaces

For Example:
- dimension
- location
- attributes
- transformation



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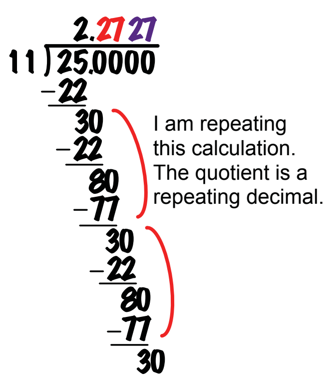
Look for and express regularity in repeated reasoning

I can notice when calculations are repeated. Then, I can find more efficient methods and short cuts.

For example: $25 \div 11$



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Handout 8: Owning the Mathematical Practices

How does Ms. McPhillips help her students "own" the math practices?
Why is it important to use the language of the Common Core with students?
How does Ms. McPhillips make discussion about the math practices a part of her daily routine?