Mathematics Improvement Network

Mathematical Practices

Helping students to develop
the practices in their math lessons

Workshop Outline

- The Common Core State Standards for Mathematical Practice
- Processes in the Mathematical Practices
- Looking at Tasks
- Engagement Strategies
- Empowerment Strategies
- Supporting Students in the Classroom

The Common Core State Standards for Mathematical Practice

Common Core State Standards for Mathematical Practice

MP1: Make sense and persevere in solving problems.

MP2: Reason abstractly and quantitatively.

MP3: Construct viable arguments and critique the reasoning of others.

MP4: Model with mathematics.

MP5: Use appropriate tools strategically.

MP6: Attend to precision.

MP7: Look for and make use of structure.

MP8: Look for and express regularity in repeated reasoning.

(http://www.corestandards.org/Math/Practice/)
College and Career Readiness Standards for Mathematics

MP1: Make Sense and Persevere in Solving Problems

- Explain a problem and look for ways it can be solved
- Analyze a problem, make conjectures and plan a solution strategy
- If you don’t succeed, try another way

MP 2: Reason Abstractly and Quantitatively

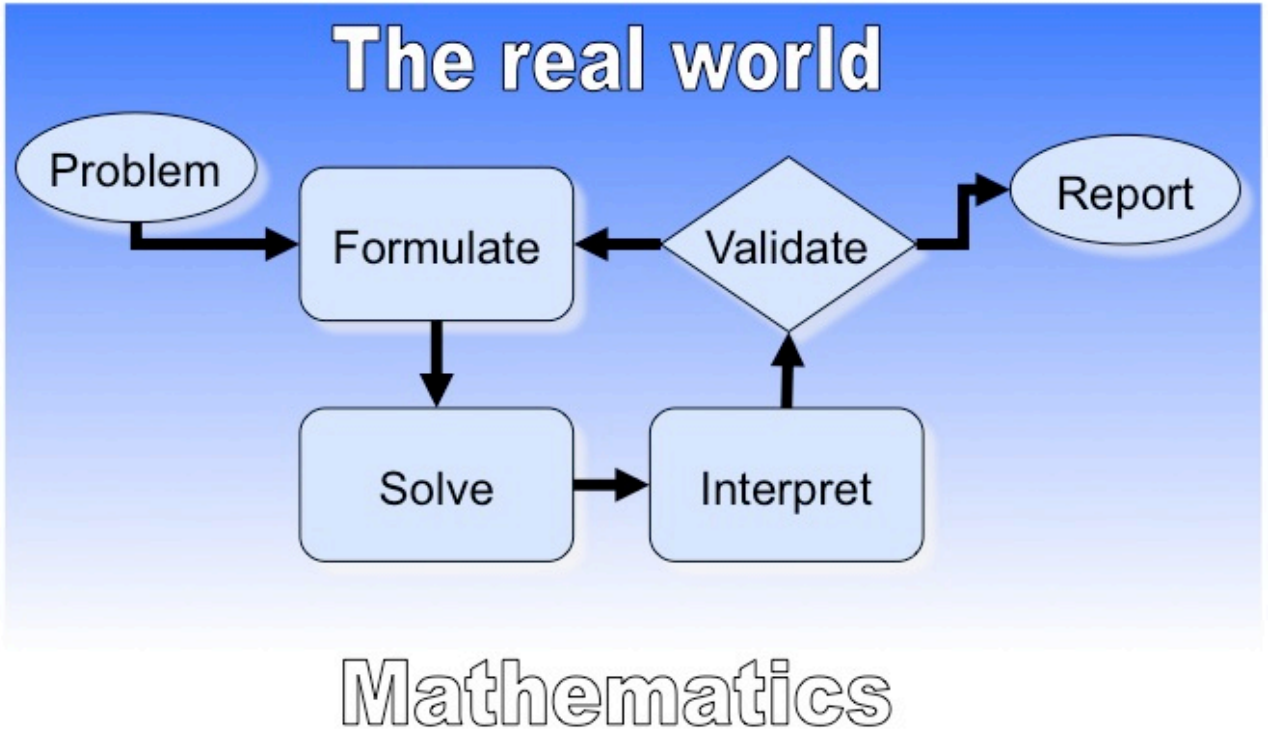
- Make sense of quantities and their relationships
- Argue coherently and use symbols to represent mathematical situations
- Use properties flexibly

MP3: Construct Viable Arguments and Critique the Reasoning of Others

- Use definitions and previous results in constructing arguments
- Justify conclusions using examples
- Distinguish between correct and flawed reasoning

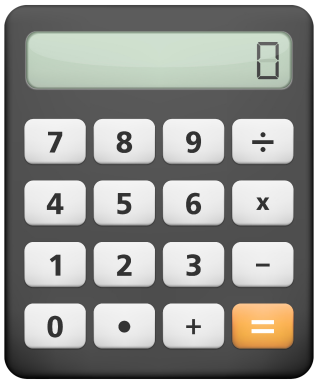
MP4: Model with Mathematics

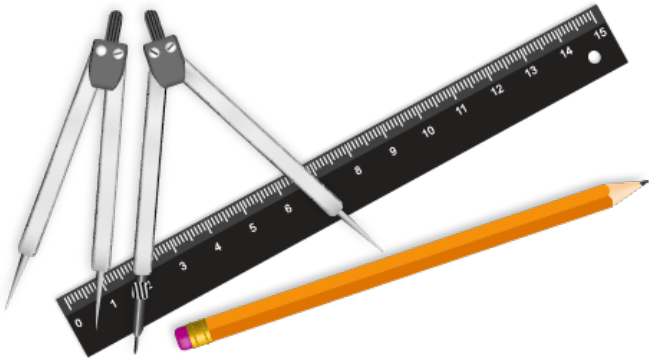
- Apply learned mathematics to everyday life situations



MP5: Use Appropriate Tools Strategically

- Consider tools available when solving a problem
- Identify helpful tools and use appropriately



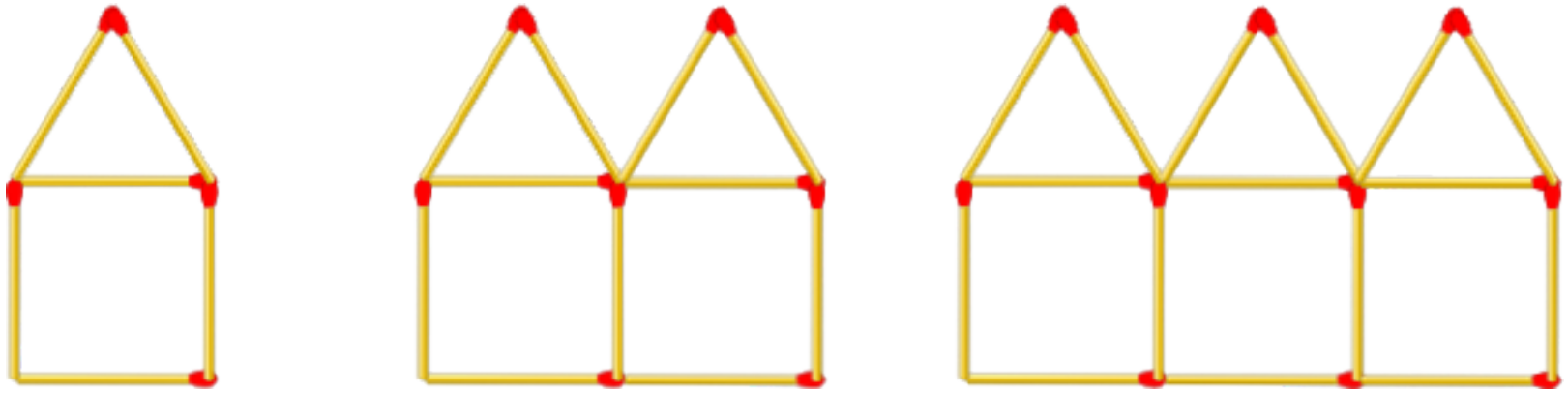


MP6: Attend to Precision

- Communicate what has been learned
- Use mathematical definitions to explain reasoning
- Show precision

MP7: Look for and Make Use of Structure

- Discern patterns and structures in math



MP8: Look for and Express Regularity in Repeated Reasoning

- Recognize when calculations are repeated and look for shortcuts
- Check on the right track when working through math problems

Processes in the Mathematical Practices

Processes in the Mathematical Practices

Problem Solving

MP1: Make sense of problems and persevere in solving them
MP6: Attend to precision

MP2: Reason abstractly and quantitatively
MP3: Construct viable arguments and critique the reasoning of others

Communication, Reasoning, and Proof

MP4: Model with mathematics
MP5: Use appropriate tools strategically

Representations

MP7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning

Making Connections and Generalizing

(adapted from 'Grouping the practice standards', William McCallum, The University of Arizona)

Problem Solving

Problem Solving

Problem Solving

MP1: Make sense of problems and persevere in solving them
MP6: Attend to precision

- Solving problems that arise in mathematics and in other contexts
- Applying and adapting a variety of appropriate strategies to solve problems
- Building new mathematical knowledge through problem solving
- Monitoring and reflecting on the process of mathematical problem solving

(adapted from www.nctm.org/Standards-and-Positions/Principles-and-Standards/Process/)

Communication, Reasoning, and Proof

Communication, Reasoning, and Proof

MP2: Reason abstractly and quantitatively
MP3: Construct viable arguments and critique the reasoning of others

Communication, Reasoning, & Proof

- Organizing mathematical thinking through communication to peers, teachers and others
- Using math language to express mathematical ideas precisely
- Making and investigating conjectures and developing mathematical arguments and proofs
- Selecting and using various types of reasoning and methods of proof

(adapted from www.nctm.org/Standards-and-Positions/Principles-and-Standards/Process/)

Representations

Representations

MP4: Model with mathematics
MP5: Use appropriate tools strategically

Representations

- Creating and using multiple representations to organize, record, and communicate mathematical ideas
- Selecting, applying, and translating among mathematical representations to solve problems
- Using representations to model and interpret physical, social, and mathematical phenomena

(adapted from www.nctm.org/Standards-and-Positions/Principles-and-Standards/Process/)

Making Connections and Generalizing

Making Connections and Generalizing

MP7: Look for and make use of structure

MP8: Look for and express regularity in repeated reasoning

Making Connections & Generalizing

- Recognizing and using connections among math ideas, as well as in contexts outside of mathematics

- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole

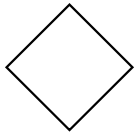
(adapted from www.nctm.org/Standards-and-Positions/Principles-and-Standards/Process/)

Looking at Tasks

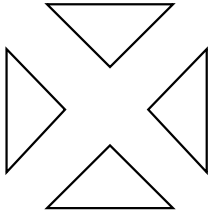
Table Tiling

Maria makes square tables, then sticks tiles to the top.

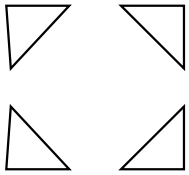
She uses three types of tiles:



whole tiles



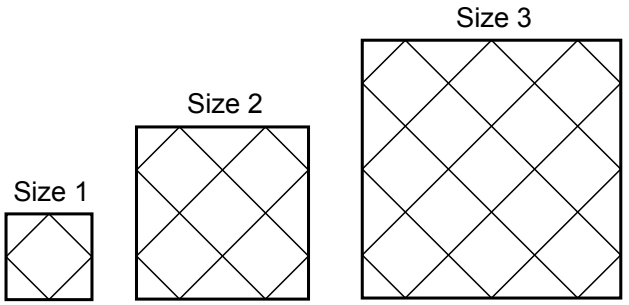
half tiles



quarter tiles

Maria only uses quarter tiles in the corners and half tiles along the edges of the table.

Here are four table tops:



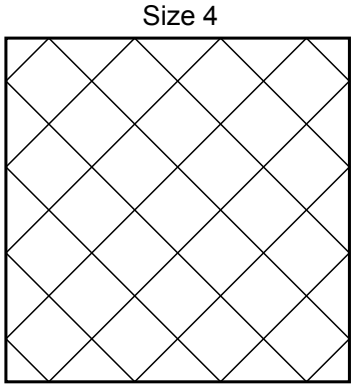


Table Tiling

1. Complete this table to show how many whole tiles, half tiles, and quarter tiles she needs for each of these sizes.

Size (n)	Number of whole tiles (w)	Number of half tiles (h)	Number of quarter tiles (q)
1	1	0	4
2	5	4	
3	13	8	
4			

2. Find a rule, or a formula, that will help Maria figure out the number of half tiles that she needs for tables of different sizes. Explain how the rule works.
3. Use the number patterns in the table to find a rule, or a formula, that will help Maria figure out the number of whole tiles Maria needs for tables of different sizes. Explain why your rule works.
4. Maria has made a table with 20 half tile. How many whole tiles are on this table? Show how you found the number of whole tiles.

Identifying Mathematical Practices

MP1: Make sense and persevere in solving problems.

MP2: Reason abstractly and quantitatively.

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Engagement Strategies

Engagement Strategies

- Initiating Pair-Share
  (or Think-Pair-Share)
- Showing Thinking in classrooms
- Questioning and Wait Time

Pair-Share

- Give students a couple of minutes to think and work with a partner, before requesting an answer to the question/ problem set

Think-Pair-Share

- Students are given time to think independently before working with a partner

Showing Thinking

- Challenging, open ended problems to reveal student thinking

- Higher degrees of student involvement with the expectation for students to express their thinking in detail

‘Every Pupil Response’ strategies:

- Thumbs up/down
- Individual white boards

Questioning and Wait Time

- Use of thought provoking questions
- Wait time that allows students to think and work towards an answer

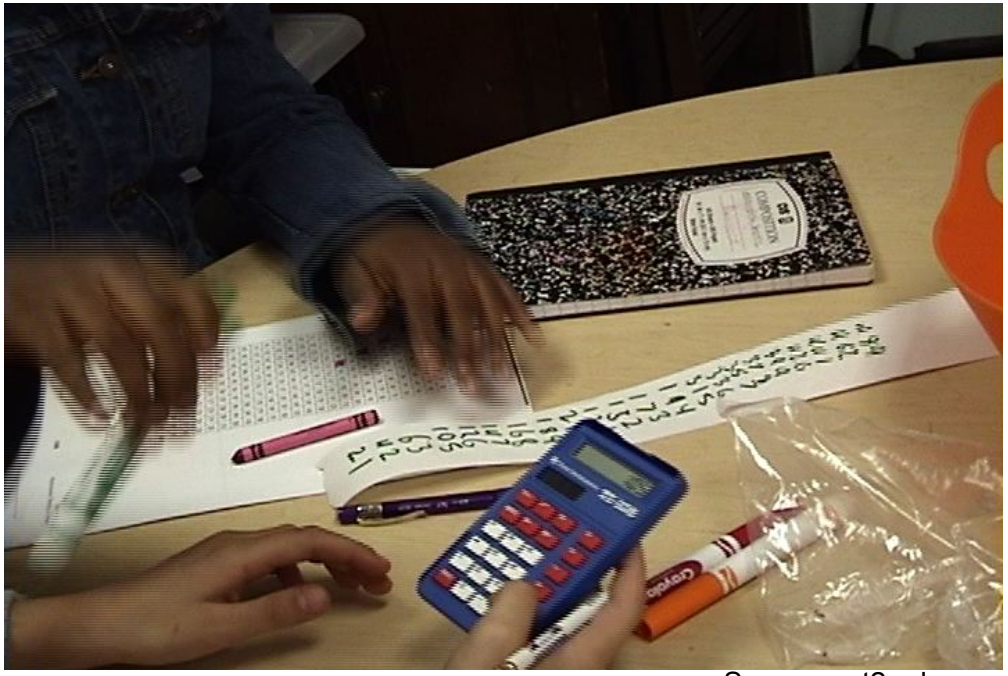
Empowerment Strategies

Empowerment Strategies

- Grouping & Engaging Problems
- Using Questions & Prompts with Groups
- Allowing Students to Struggle
- Encouraging Reasoning

Grouping & Engaging Problems

- Challenging problems
- Groups of 2, 3 or 4



Source: cct2.edc.org

Using Questions & Prompts with Groups

- Increased teacher ability to ask supporting questions
- Give hints without telling students the answer
- Probing questions

Allowing Students to Struggle

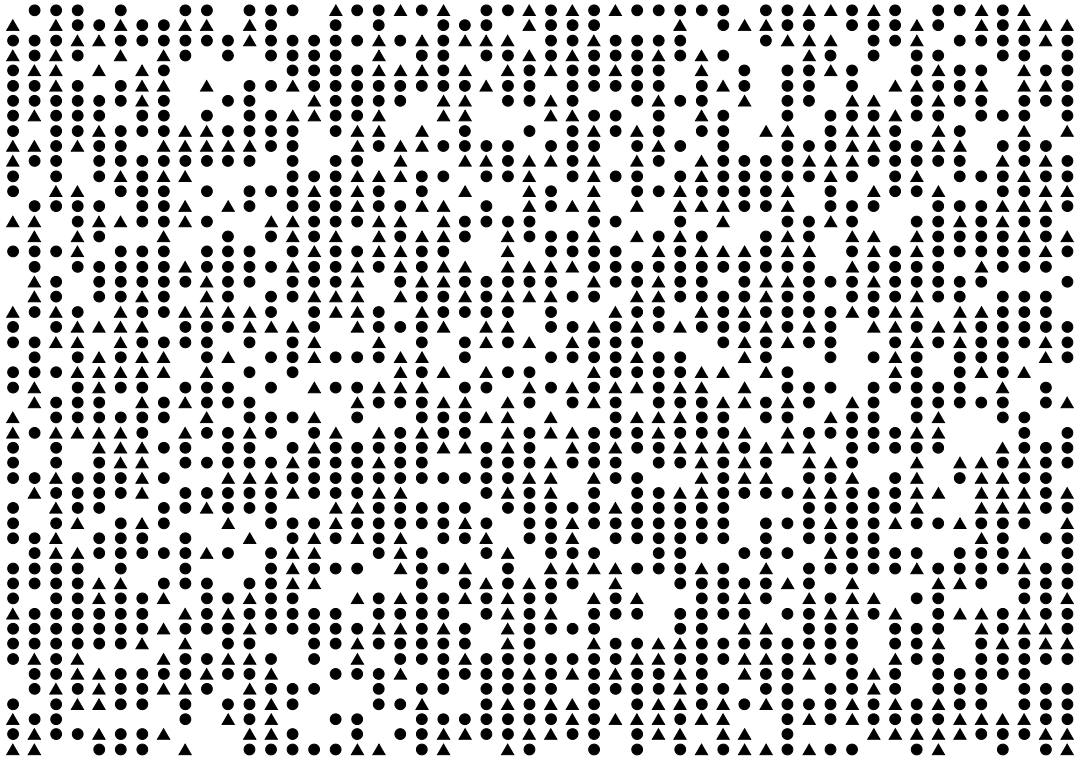
- Providing opportunities for students to learn to persevere
- Appropriate degree of difficulty
- Balance between group work and independent work

Encouraging Reasoning

- Understand their level of knowledge
- Communicate their careful thinking about mathematics
- Applying patterns, connections and rules of mathematics to finding a solution

Sampling & Estimating - Counting Trees

The diagram shows some trees in a tree farm.

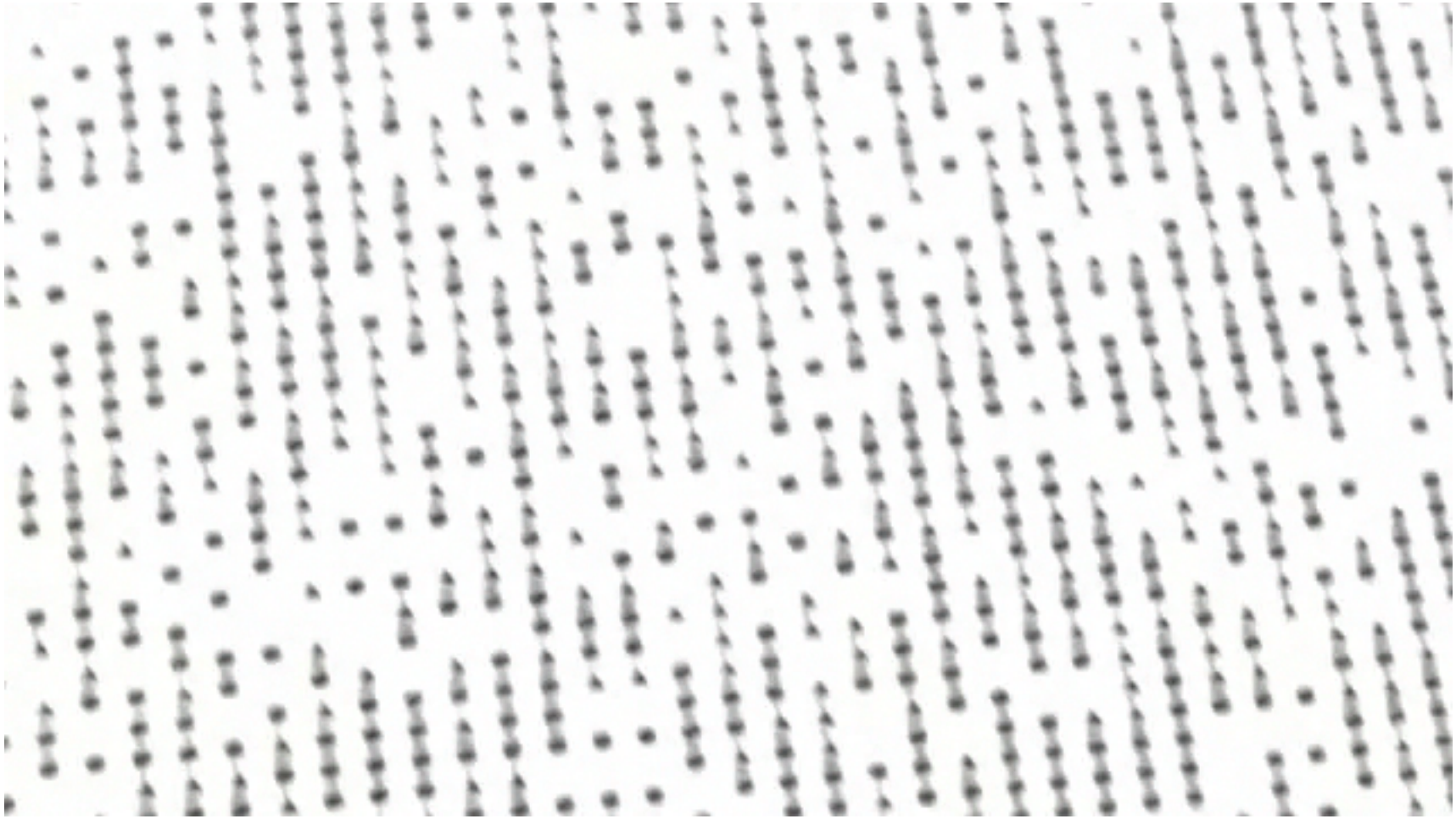


The circles • show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one by one.

A lesson based on this task can be found at http://map.mathshell.org/lessons.php?unit=7400&collection=8

Tree Counters



Source: https://mathisnotacrime.wordpress.com/math-practices-video-series/

Supporting Students in the Classroom

Interpreting the Mathematical Practices

Mathematical Practice	Student-friendly language
Make sense & persevere in solving problems	I don't mind trying many times to understand and solve a math problem.
Reason abstractly & quantitatively	I can think through a problem step by step, using numbers, graphs and math symbols, checking back with the problem each time.
Construct viable arguments & critique the reasoning of others	I can explain how I solved the problem and why my reasoning is correct. I can discuss other students' solutions too.
Model with mathematics	When faced with a real-world problem, I can make sense of it, then use pictures, numbers, tables, graphs, and symbols to represent it and work out a solution.
Use appropriate tools strategically	I can choose the right math tool for solving a problem: like calculators, rules, pictures, or objects.
Attend to precision	I can make sure my method and my calculations are correct and my explanation is complete and convincing, using clear mathematical language.
Look for and make use of structure	I can use what I already know about math to solve the problem, using equivalent expressions to make sense of things.
Look for and express regularity in repeated reasoning	I can look for a pattern in a problem, whenever the reasoning repeats. I can represent the pattern by a rule or formula.

Adapted from: Teaching Children Mathematics, March 2012, *Getting Started in K – Grade 2*, pg 440-445

Resources to Support Students

Make sense of problems and persevere in solving them

When presented with a problem, I can make a plan, carry out my plan, and evaluate its success.

BEFORE...

EXPLAIN the problem to myself.

- *Have I solved a problem like this before?*

ORGANIZE information...

- *What is the question I need to answer?*
- *What is given?*
- *What is not given?*
- *What are the relationships between known and unknown quantities?*
- *What tools will I use?*
- *What prior knowledge do I have to help me?*

DURING...

PERSEVERE

MONITOR my work

CHANGE my plan if it isn't working out

ASK myself, "Does this make sense?"

AFTER...

CHECK

- *Is my answer correct?*
- *How do my representations connect to my algorithms?*

EVALUATE

- *What worked?*
- *What didn't work?*
- *What other strategies were used?*
- *How was my solution similar to or different from my classmates'?*

Source: http://www.debbiewaggoner.com/uploads/1/2/9/9/12998469/math_practices_posters_ms_hs.pdf

Resources to Support Students

Look for and express regularity in repeated reasoning

I can notice when calculations are repeated. Then, I can find more efficient methods and short cuts.

For example: 25 ÷ 11

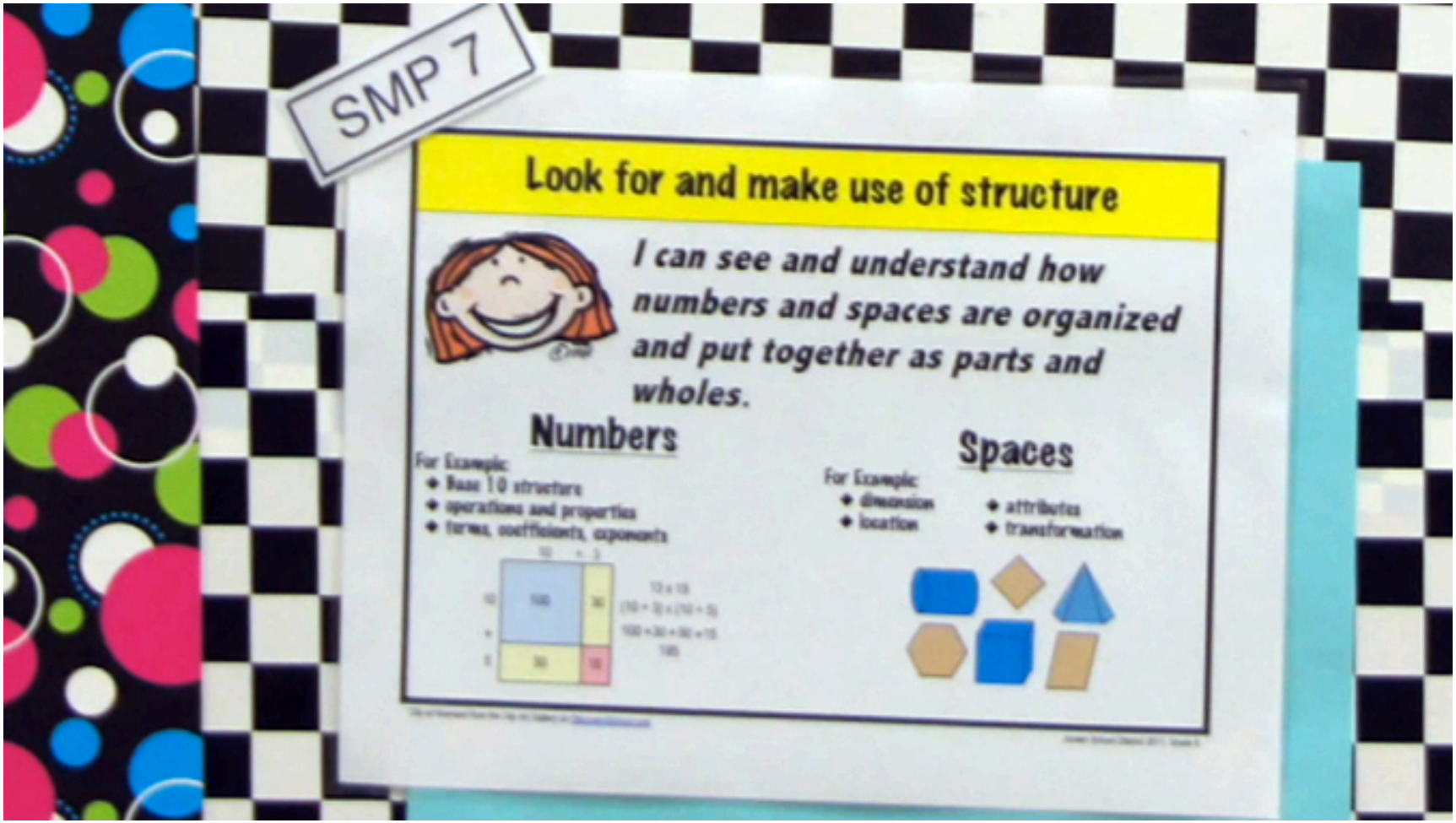
$$
\begin{array}{r}
2.2727 \\
11\overline{)\,25.0000} \\
-22 \\
\hline
30 \\
-22 \\
\hline
80 \\
-77 \\
\hline
30 \\
-22 \\
\hline
80 \\
-77 \\
\hline
30
\end{array}
$$

I am repeating this calculation. The quotient is a repeating decimal.

Owning the Mathematical Practices

- How does Ms. McPhillips help her students “own” the math practices?
- Why is it important to use the language of the Common Core with students?
- How does Ms. McPhillips make discussion about the math practices a part of her daily routine?

Owning the Mathematical Practices



Source: https://www.teachingchannel.org/videos/owning-the-common-core

In Summary

- Talking about the practices enables them to be part of students' common language
- Students should be involved in the process of tracking their practices as they tackle rich problems
- Identifying tasks that provide opportunities for the development of each of the eight practices promotes a balanced curriculum

Mathematics Improvement Network

Thank you

< insert contact details >