Program Coherence Health Check
How well do our various drivers support policy for mathematics?

Presenter's Guide

GOALS
This tool helps district leaders to review the alignment of the various parts of their mathematics program, to answer the question: Do our curriculum (instruction materials), assessments and professional development offerings support policy and the standards?

USERS
District leadership. The coherence challenges for individual schools are similar in many ways, so this tool may also be useful for principals and their subject leaders.

INTRODUCTION
What is program coherence? Student progress is largely determined by what happens in classrooms, where the teacher is the key mediator. Programs drive, constrain, motivate and support teaching through our policies, curriculum, assessments, professional development and leadership choices. Are these drivers of teacher action coherent with one another? If not, we pay a double penalty:

1. The program works against itself, tripping over its own feet, resulting in wasted effort and weak performance.
2. Personnel develop cynicism about the program and its leadership resulting in disengagement from the program.

How can we check on our program coherence?

This workshop will help leaders focus on the coherence of what reaches the classroom and drives learning. The key to understanding the realities of coherence and practical steps for improving program coherence is this: focus on what students do, what teachers do to help students learn from what they do and what the program does to guide and support teachers in a coherent direction.

One essential indicator of standards is the range of task types that students work on. If students spend most of their time doing low-level tasks, their achievement will be low level. High standards depend on putting the right variety of tasks in the assessments, the curriculum and the professional development. Of course, there is much more to a fine program than this but the range and balance of task types is a reliable probe - "the canary in the coal mine".

The diagnostic approach in this session is based on the leadership comparing a sample of the types of mathematical tasks found in the assessments and curriculum that the district uses with those used in high-performing jurisdictions, and described in the Common Core.

This 'Program Coherence Health Check', in the form of a 90-minute workshop, is the first step. 'Ways to Improve Program Coherence' is a follow-up tool that provides guidance on improvement options in any of the main action areas that this health check may suggest needs attention.
SESSION OUTLINE

- What is program coherence?  
- Key diagnostic probe: the range of task types  
- Expert, Apprentice and Novice Tasks  
- Looking at our program (the core, your prep - don’t get here late)  
- Different kinds of math challenge  
- What have we learned? - a forward look

estimated times 5 minutes  
5 minutes  
25 minutes  
30 minutes  
10 minutes  
10 minutes

MATERIALS

- This Presenters Guide, along with the PowerPoint: ‘Coherence Health Check slides.pptx’  
- Session Handouts: One copy per participant, in the form of a booklet.

NOTE: Handouts 4 and 5 need to consist of sample tasks from the district’s own curriculum and assessment respectively. These must be prepared in advance, by the district's mathematics leadership. We suggest focusing on Grade 6, at least for the first time - though another grade could be chosen, depending on the mathematical level of the workshop participants.

TIME NEEDED

90 minutes.

PREPARATION

To give district leadership a clear and valid view of program coherence in a single workshop, a significant amount of preparation by the district's mathematics team is needed, as follows.

1. The workshop leader should carefully work through this guide with the PowerPoint slides, which contain most of the same material, including the suggestions and comments with each slide. You may like to fill in your local information on the first and last slides.
2. Handouts 4 and 5 need to be created. When you have absorbed what the session involves, select for Handout 4 samples of the three types of task (Expert, Novice, and Apprentice – described below) from the district's most widely used curriculum materials. It is likely you will need to take samples from a few different chapters or units to show the full variety of task types. You may find it convenient to copy each sample task into a table, as in the Model for Handout 4 and 5, given at the end of the Handouts document - delete this model before printing the handouts for participants.
3. Handout 5 is similarly selected from the assessments used in the district. The goal is to show the range and balance of task-types involved in the district program's assessments and classroom activities.
4. As you do this selection, make a rough estimate of the proportions of student time spent working on each type of task, in the curriculum and in the assessments. Record these estimates on Handouts 4 and 5, as shown on the model. [Ideally, also modify the pie charts in PowerPoint slides 26 and 28. (Click on the pie chart, the Charts button and Edit button. This will allow you to change the data represented). Otherwise, just say these pie charts are symbolic.]
5. Try to anticipate the common issues that participants will have and note down your responses to them, below. The ones shown below are examples taken from trials of this session.
6. To support productive discussions in the session, we suggest that you pair people with a STEM background with those from other academic areas.
<table>
<thead>
<tr>
<th>Common concern</th>
<th>Suggested responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why this focus on tasks? There's much more to learning mathematics than tasks.</td>
<td>True, but the range of task-types is a clear but simple indicator of performance goals - &quot;a canary in the coal mine&quot;. A broad and balanced range is not sufficient for high standards but it is necessary, indicating the opportunities to learn and perform.</td>
</tr>
<tr>
<td>A lot of these expert tasks aren't math; they're about everyday life.</td>
<td>True but the ability to use (often elementary) math effectively in thinking about challenges in everyday life is: a major reason for the amount of time allotted to math; a major focus of PISA, the Common Core and related State Standards; increasingly important for life in the 21st Century. Most students, if they don't learn these skills in math class, won't ever learn them. Real life problems also motivate many students who are not turned on by math - or haven't learned to accept (long) deferred gratification.</td>
</tr>
<tr>
<td>What if a student, or a teacher, doesn't know the context? We can't have biased tasks.</td>
<td>First, research shows that all tasks are biased - bare novice tasks included. Good design seeks contexts that are widely understood by students - and a balance of any residual bias across different groups (including white, middle class males). You can decide how well the tasks you see in this workshop exemplify this.</td>
</tr>
</tbody>
</table>
**ACTIVITY SEQUENCE**

*Revised version: Summer 2016*

**Setting the scene**

(5 minutes)

You may like to customize this slide and/or the last one with your own institutional and contact details.

*Please leave the copyright attribution, however.*

Possible comments below are in plain text. Suggestions are in italics.

Today’s meeting is about engaging in a conversation on the coherence within the district mathematics’ program; asking ourselves if the various aspects of the districts’ program are well-aligned?

Investigating this is our goal in this workshop.

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**Move through the first 10 slides fairly quickly**

Posted here, is an outline of what we are going to work through today in our effort to make sense of the coherence of current district (school) mathematics programs.

**Rough timing**

- What is program coherence? 5 minutes
- Key diagnostic probe: the range of task types 5
- Expert, Apprentice and Novice Tasks 25
- Looking at our program 30
- don’t get here late
- Different kinds of math challenge 10
- What have we learned? - a forward look 10

This slide explains what we mean here by program coherence.

*Read the slide slowly*

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**Workshop Outline**

- What is program coherence?
- Key diagnostic probe: the range of task types
- Expert, Apprentice and Novice Tasks
- Looking at our program
- Different kinds of math challenge
- What have we learned? - a forward look

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**What is program coherence?**

A coherent mathematics program:

- is a focused, logical, clearly articulated set of concepts and skills
- with connections to each other and to applications in a variety of contexts from the real world and within mathematics.
- is uniform across buildings and the district providing all students the opportunity to learn important mathematics
- is supported by all aspects of the districts program.
When determining if a district program is coherent, one needs to consider the various elements that drive the program.

All programs in schools are comprised of policy statements, curriculum materials, assessments, the actual instruction that takes place in classrooms, and professional development offerings the educators are asked to attend.

To guide teachers in their instruction within classrooms, one needs to ask-- are the messages to teachers, across these domains, consistent and aligned?

For example, it is so easy for poor high-stakes assessment to undermine the intentions of our policy statements and curriculum materials and thus the instruction in the classroom.

If any element is out of line, the whole program can fail.

How would you describe the underlying purpose and goals of your school mathematics programs? The statement posted, from the Common Core State Standards draft, reflects the beliefs, purpose and goals found in high-performing countries, per PISA, and the mathematical needs of society for the 21st century.

As you read the slide, ask yourself:

Is it what all of us in our district are working towards?

(The quote is taken from a draft of the common core document)

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**Key diagnostic probe**

(5 Minutes starting at ~5 minutes)

A powerful probe one can use to think about the coherence within a program is to review and analyze the **variety of types of task** that students are asked to tackle.

This strategy is based on examining the types of task in the program's curriculum materials and assessments.

If students spend most of their time doing low-level tasks, their achievement will be low level. The key to higher standards, and better mathematics programs, is improving the tasks that students work on most of the time.

**Tasks are key indicators of coherence – they are the canary in the coal mine.**
Coherence means having the right kinds of tasks in the assessments, the curriculum materials and the PD offerings; tasks that together create a balanced mathematical experience for students and teachers. It also means having policies that support ALL students in ALL classes using a balanced set of curriculum materials and receiving instruction with a range of task types.

Basically, by reviewing and analyzing tasks, one can get a better understanding of how policy, assessments, curriculum materials and PD offerings, linking together and the kind of coherent, or incoherent, message that is being sent by the district.

A coherent program is not only a focused and clear set of concepts, skills and strategies to be developed over time, but it also progresses in complexity.

In English Language Arts progression is measured by the student’s ability to deal with increasingly complex texts, in increasingly sophisticated ways.

In high-quality Mathematics curricula it is similar but with tasks instead of texts

Read the “In Mathematics” paragraph
This may be a different way of looking at progression in math. It’s not just moving on from arithmetic to algebra but also the complexity of the situations in which one needs to use mathematics to solve problems.

This holistic view is standard in ELA – and in mathematics outside school.

Thus, coherent mathematics programs involve learning mathematical content but also, and equally important, being able to use the content to solve problems.

As you read the slide, note the two aspects

• Learning concepts and skills is like building a toolkit. This is, of course, vitally important…

• .. but students also need to learn how to select from and use these tools to tackle problems from within mathematics and from the real world.

This is what high standards are all about.

It’s no good building up a really sophisticated toolkit and set of skills, if you cannot use them to make anything.
Read the slide slowly with emphasis

The point being—one needs different kinds, different types of task to be sure that one has a coherent, balanced program that is preparing students to be the “proficient student” for whom

• mathematics makes sense and who

• can take an active approach in solving problems that require mathematics.

That is: a program that includes students doing tasks that show their knowledge of the content but also tasks that show they can use the components in their mathematical tool kit to solve problems and reason.

Expert, Apprentice and Novice Tasks

(25 minutes starting after 10 minutes) 1
including time for discussion of the problem on Slide 18 - Airport Turnround

What do we mean by "types of task"?

It is useful to distinguish three very different types of task: Expert, Apprentice and Novice

Descriptions of each type are given in Handout 1—it's not needed right now but will be given out later.

First, let's see some examples of each type.

Later we’ll analyze each kind and how they differ.

Move through the 5 novice task slides fairly quickly, pausing on those that best fit the math level of the participants.

We'll start with some novice tasks, because these are most familiar in school math.

Each task involves a standard routine procedure - one that students have been taught and, hopefully, learned.

Even though the techniques become more complex, these are all imitative: novice tasks. Tasks of this type are also referred to as “low-level” tasks in the literature—though they can, of course, be difficult.

Pause for a moment, then move on.
Here's another one for elementary school - a novice task on perimeter.

Even though there are “words” in this problem, this is still a novice task as it is just using the perimeter definition and a learned procedure for calculating it.

Optional slide. If participants do not include high school mathematics teachers consider not using this slide.

Here's another one - about graphs of functions

Only short chains of reasoning are required, only a few steps – a key feature of novice tasks.

So which graph is which function?

Ask for suggestions, then

Can you explain why?

Here's one for middle school

Often the easiest way to solve a multiple choice task is by elimination - not by doing the math.

That can lead to "test prep" time that improves scores but not the students' math.
Optional if participants do not include middle or high school math teachers.

Another multiple choice example

**Explanation paragraph optional – for more mathematical groups**

Here the concepts of linearity, growth and decay are enough to solve this task.

Linear means equal steps in x mean equal steps in y – as in the first one

Of the other two one increases – growth – the other goes down – decays

You don’t actually need to understand exponentials: equal steps in x mean equal ratios in y

**Now lets look at some expert tasks**

What is an expert task? *Read the bullet point paragraphs*

Because expert tasks are less familiar in school math, let’s look at a few examples - for middle school.

Expert tasks, like essay topics, are asking students to put together multiple ideas and use them in complex situations.

These tasks are less grade-specific, allowing students to respond at their level. They reveal to educators what tools from their toolkit of concepts and skills students can actually use in problem solving and reasoning.

The tasks being shown are from high-stakes tests, not commonly used in then US, but used regularly in other countries’ tests.

**Take time for discussion on this slide**

Airplane Turn-round is quite demanding even though the necessary content is just simple arithmetic.

Guess what the kids tend to do? *If no response* Yes, they just add the numbers. That’s school math.

But when you ask “Is there a quicker way?”, they soon get to talk about what things can be done in parallel. You might like to talk with your neighbor about how you might approach solving it.

**Allow a few minutes for pairs to discuss, then**

What did you find? Other ideas?

*Take suggestions without comment. [The shortest turnaround time is 65 minutes = (D+F+G)]*

That’s a brief taste of what it means to engage with an expert task.
Quick slide Just talk this through

Traffic Jam is another nice example of an expert task. The problem relates to everyday life.

Read the slide

The necessary content is ratio and proportion with some knowledge of units. What makes it more demanding is that you have to identify the relevant data that is needed (things like average vehicle length, gaps between vehicles, proportions of different vehicles), make assumptions as to their value, formulate the appropriate division calculation, and interpret the result.

In part 2, it's reaction time times the number of cars.

Quick slide Just talk this through

Here’s another nice example

Read the slide

Counting trees involves coming up with a strategy for sampling that incorporates the relevant facts and is represented in a suitable way. Students need to make clear the assumptions they have made and the effect these assumptions have on the answer.

These three expert tasks show something of the nature of mathematical expertise, described in the Mathematical Practices.

Read the slide

Look at Handout 2: ‘Multiplying cells’

Talk it through it with your neighbor.

Leave a minute or two for this

Notice how specific design tactics are used, “scaffolding” the essential task into a sequence of prompts.

• The diagrams, and the number of cells by counting, ease the entry to the problem.
• The structuring into parts reduce the strategic demand.
• Providing the table directs the representation aspect.
• There is a steady increase in difficulty through 1-5.

Having a lot of experience with apprentice tasks like this helps students develop the habits of expertise they need to use autonomously, without “the guide”, in tackling expert tasks.

An Expert Task

Traffic Jam
1. Traffic has come to a stop. The line is 12 miles long on a two-lane freeway. How many cars are in the traffic jam?
2. Drivers have a two-second reaction time. When the accident was cleared, how long for the last car to move?

An Expert Task

Counting Trees

The diagram shows some trees in a tree farm. The circles show old trees. The triangles show young trees.

• Think of a method you could use to estimate the number of trees of each type. Explain the method fully.
• Use your method to estimate the number of old trees and young trees.

Apprentice tasks

• Expert tasks with added scaffolding to:
  – ease entry
  – reduce strategic demand
• Ramp of difficulty within the task, with increasing:
  – complexity
  – abstraction
  – demand for explanation
• A step in growing expertise: “climbing with a guide”
Option: if the Cells task seems too advanced for elementary school folk, this or the following slide could be used for discussion instead of Handout 2

Here's an apprentice task for elementary school

I have some pennies, nickels, and dimes in my pocket. I pull out three of the coins and keep them in my hand.

a. How much money do you think I have in my hand? Find as many different amounts as you can think of and list on your paper.
b. Could I have 11¢? How, or why not?
c. Could I have 4¢? How, or why not?

Here's an apprentice task based on elementary school or middle school math - depending on which method student decides to use.

An option at middle or high school math level
Scaffolding can be more subtle than what is outlined in 'Multiplying cells' (or the more elementary Coin Problem). In 'Skeleton Towers'

- 6 cubes high can be worked out by counting
- 20 cubes high invites a solution based on a pattern: (e.g. 1 + 5 + 9 + 13 + .... + 77)
- n cubes high invites a complete generalization: \(n(2n-1)\). This may be obtained by various methods, algebraic or geometric.

Without the first two prompts, this would be an expert task
If you have plenty of time, and a group interested in math, you may like to discuss different possible solution approaches. Perhaps the simplest way is to break off two opposite legs, turn them upside down on the other two - and so make a rectangle!
Now let’s compare the three tasks in Handout 3.

The three have the same underlying math but present very different kinds of challenges.

Discuss with your neighbor where difficulty might arise in each task. Decide, for each version, if you think they are a novice, apprentice or expert task.

Allow at least 5 minutes, then collect comments. Summarize (or reflect) as follows.

**Patchwork Version 1** is a task in a situation that might well arise. You have to work out the strategy and do the whole thing, formulating the rules. This strongly involves the mathematical practices. It’s an Expert task.

**Version 2** gives a formula, asks students to apply it, then solve an inverse problem. This is a routine exercise. It’s a Novice task.

**Version 3** breaks the task in Version 1 into a series of guided steps. It’s an Apprentice Task.

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**Looking at our program**

(30 minutes starting after ~35 minutes.)

*This section is the heart of the workshop – feel free to extend it if there is more time.*

Now we’re going to use this same “task review and analyze” approach to look at and classify the messages we send to our teachers and students:

- first – through curriculum;
- then – through assessment.

At this time, we will not be examining professional development offerings or district policies – but focus on discussion of the evidence from two components of our program, and the implications.

Just a reminder of this summary of the goals of mathematics education.

*Point again to a few key phrases: make sense, plunge in and try, carry through procedures, think strategically.*

Now let’s look at our system.

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**Success in Mathematics**

“Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.”
Say which grade the materials are from
Add that the sample is probably fairly typical across grades – if you think so
Give them plenty of time to look through the tasks and consider your labels E, A and N
Discuss any questions.
Now to core questions:
What is the balance?
Is it reasonable?
Go straight to next slide

As you make the sample for Handout 4, estimate the proportions across the year's curriculum
These are numbers for you to enter on the Handout (and the slide if you can revise it! See Preparation)
You will see at the end of the Handout that the times students spend in our curriculum are approximately Expert xx%, Apprentice yy%, Novice zz%

Explain the last 2 points more fully
The teaching for expertise that the Common Core requires is more demanding on teachers, and includes aspects that are new to many.
Will they think that they can afford to ignore this amount and just continue with novice math?

Read the slide
This activity should replicate that for curriculum in the last two slides, but now with Handout 5.
Again, say which grade the materials are from
Add that the sample is probably fairly typical across grades – if you think so
Give them plenty of time to look through the tasks and consider your labels E, A and N
Discuss any questions.
Again, the core questions
Go straight to next slide

Does our curriculum develop expertise?
Handout 4 is a sample of the tasks in the curriculum our students go through.
• We have labeled each task E, A or N
  Take a few minutes to review our classifications with your neighbor.
The core questions:
• What is the balance across E, A and N?
• Is it reasonable?

A balanced diet?
• We have looked at our curriculum and estimated the proportions of classroom time that our students spend on Expert, Apprentice and Novice tasks. The numbers are at the end of the Tasks Handout
• Are these proportions all substantial?
• Are they what we want?
• Does the curriculum support teaching for expertise?
• Could teachers ignore teaching for expertise?

Do our tests support expertise?
Handout 5 is a sample of the tasks in the by which we judge our students and teachers.
• We have labeled each task E, A or N
  Take a few minutes to review our classifications with your neighbor.
Again, the core questions:
• What is the balance across E, A and N?
• Is it reasonable?
As you make the sample for **Handout 5**, estimate the proportions of time across the year’s assessments

**These are numbers for you to enter on the Handout (and the slide if you can revise it! See Preparation)**

You will see at the end of the Handout that the times students spend in our curriculum are approximately Expert xx%, Apprentice yy%, Novice zz%

Encourage participants to discuss the questions on the slide be sure to ask the last question--

What has been shared with you is a rough estimate of what we found when we looked at the curriculum materials and district assessments.

Are you happy with the “coherence” presented in these two domains of our program? Do they reflect the same message that we have sent to teachers and principals through out professional development offerings and district policies? Will teachers and principals think that they can and should continue with novice tasks– because it won’t reduce their students’ scores on these tests by much?

As you continue to think and reflect on the district program, lets think about why is it important to have these different types of task. Each of these task types has a different role. 

Note the two aspects of knowing mathematics:

Learning concepts and skills is like building a toolkit. This is of course vitally important…

.. but students also need to learn how to select and use these tools to tackle problems from within mathematics and from the real world.

This is what problem solving and reasoning, and the mathematical practices, are all about.

It’s no good building up a really sophisticated toolkit, if we never use them to make anything.

Let's look at this in a bit more detail.
What makes a math task difficult?
(10 minutes starting at ~ 65 minutes)

What makes a math task difficult?
This is a key issue.
We are used to thinking it is just "harder math", more advanced content. But with the broader range of task-types that high standards involve, that is only part of the story.
Let's look at it.

This is fairly obvious

- If a task is made more complex, it gets harder.
- If it needs advanced content, it’s harder – we’ve always known that and, perhaps, only that.
- If you’ve been taught how to do one very like it, it’s easier.
- Constructing a long chain of reasoning is harder than responding to a scaffolded sequence of prompts.

But, though this is obvious, it has profound implications because the Standards, like the 21st century world, value problem solving and reasoning. This means students tackling complex, non-routine tasks.
Solving them demands long chains of reasoning.

Read the sentence below the diagram

This means that complex, non-routine tasks should not require recently-learned content.

Such tasks - the right side of the balance - complement technical exercises on grade level content (the left side)

Mathematical literacy, which is what PISA assesses, is “The sophisticated use of [relatively] elementary mathematics”

Quote from Lynn Steen in "Mathematics and Democracy".

Tasks become more difficult as you increase:

- **Complexity**
  - Are there many variables, complex information, messy situations?
- **Technical demand**
  - Are challenging concepts and skills involved?
- **Unfamiliarity**
  - Or have students met tasks just like this before?
- **Autonomy expected**
  - Are long chains of unprompted reasoning involved?

Two tasks – Similar difficulty

For two tasks with comparable ‘cognitive load’, one that is more demanding in some aspects must be less so in others
It is useful to summarize these differences and distinguish between Novice tasks, Apprentice tasks and Expert tasks. Their different kinds of difficulty are shown in this very schematic diagram – by the font sizes.

**point to diagram**

Moving from left to right

- Complexity, unfamiliarity, and student autonomy increase - along with the need for skill in the mathematical practices
- Technical demand has to decrease to maintain the level of difficulty

**Handout 1** summarizes these things – for later reading

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**What have we learned?**

*(10 minutes starting at ~ 75 minutes)*

This session has been about diagnosis – learning what our current program is like on coherence. It has given us all something to think about.

What are your initial impressions?

*Let people make their points but limit the time, moving on to the next slide*

Obviously there is a lot to talk through another time

But before we finish, let's look forward

*Read the slide*

Given this context, and where do we think we are?

Let's begin to think about strategies for improvement - clearly something to follow up on if we think it's needed.

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**Discussion points**

**Context**

- Many US classrooms have just taught "novice math" – reflecting the short-item tests on which success has been judged. New tests are better.
- Teaching math for expertise is harder, requiring teachers to gain “adaptive expertise” – the ability to modify one’s lesson to build on students’ responses.

**Action**

- If our assessment and curriculum are balanced across E, A and N tasks: fine – we can focus on developing teachers’ expertise – otherwise,…
Read the first bullet

The right balance of task-types is not sufficient to guarantee coherence (a teacher could show the students how to tackle an expert task, for example, reducing it to a sequence of novice tasks) but it is a necessary condition, an essential opportunity to learn, and to perform.

The Mathematics Improvement Network has developed a follow up session through which we can review ways to improve coherence.

Shall we move to schedule that?

We have not looked at professional development - one of the four areas over which coherence is important that we set out at the beginning.

Read the slide

In this district we have a lot of experience and some well-established professional development. But it's expensive and we don't always see the changes we would like in our classroom visits afterwards.

This MathNIC workshop is about issues in design of PD. Would we like to look at that?

Insert your contact details on this slide.
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