Program Coherence Health Check
How well do our various drivers support policy for mathematics?

Handouts

Contents

A model for Handouts 4 and 5 is given, and analyzed, at the end of this paper.
We suggest that you delete this model before duplicating the Handouts for the users.

Copying

Handout 1: Task difficulty and task types

Task Difficulty

There are four aspects of a task that contribute towards its overall difficulty:

- **Complexity**: Are there many variables, complex information, messy situations?

- **Technical demand**: Are higher level concepts and skills necessary?

- **Unfamiliarity**: Have students met tasks just like this before?

- **Autonomy expected**: Are unprompted chains of reasoning required?

Novice	Apprentice	Expert
Complexity	Complexity	Complexity
Technical demand	Technical demand	Technical demand
	Unfamiliarity	Unfamiliarity
Unfamiliarity	Autonomy expected	**Autonomy expected**
Autonomy expected		
Short items	Scaffolded tasks	Unscaffolded tasks
Up to grade skills and concepts	Ramped level of demand	Below grade skills and concepts
Few Mathematical Practices	Modestly involve Mathematical Practices	Strongly involve Mathematical Practices

Expert, Apprentice and Novice tasks of the same difficulty have different balance of these in the demand they make on students.

Expert Tasks

These are substantial tasks, as they naturally arise in mathematics or outside the classroom. They usually involve all four aspects of task difficulty, as stated above. They are complex, non-routine and unscaffolded. There is strategic demand: students must work out what to do, how to do it, and what math tools will help.

So, for comparable overall difficulty, the technical level is below grade, requiring concepts and skills that have been well-absorbed and richly connected. ["the few year gap"}

Solving expert tasks substantially involves the mathematical practices and long chains of autonomous student reasoning.

Apprentice Tasks

Expert tasks, but scaffolded into a coherent sequence of subtasks, which reduces the strategic demand and student autonomy. In good apprentice tasks, the demand increases, usually in generality and abstraction, through the sequence of parts. This "ramp" allows all students to make real progress with the underlying expert task while challenging the most able. Solving apprentice tasks involves the mathematical practices at a modest level.

Novice Tasks

These are usually short items with mainly technical demand - so they can demand "up to grade" content, including concepts and skills learned recently. Novice tasks hardly involve the mathematical practices, or assess the ability to use math on rich problems.
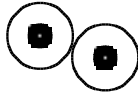
Handout 2: An Apprentice Task
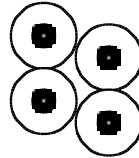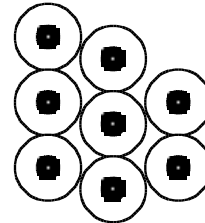
Multiplying Cells

| 0 minutes | 20 minutes | 40 minutes | 60 minutes or 1 hour |

Mrs. Lucas's class has a 2-hour science lab.
She gives each student a dish with one cell in it.
She tells the class that in 20 minutes the cell will divide into two cells,
and each 20 minutes after that **each** cell in the dish will divide into two cells.

1. Complete the second row in this table to show how the number of cells increases during the lab.

Time (minutes)	0	20	40	60	80	100	120
Number of cells	1	2	4				
Number of cells as a power of 2	2^0	2^1					

2. Olan says that the numbers of cells can be written in the form 2^n.
Complete the third row in the table to show how the number of cells can be written in this form.

3. Linda says that the number of cells after 3 hours will be 2^7 (= 2x2x2x2x2x2x2)
Is she correct?
If not, then what is the correct number?

Explain how you figured it out.

4. How many cells will be in the dish after 5 hours?
Give your answer as a normal number, not as a power of 2.
Show how you figured it out.

5. How long will it take for the number of cells to reach at least 100,000?
Give your answer to the nearest 20 minutes.
Show how you figured it out.

Handout 3: Expert, Apprentice or Novice Task?

The following three tasks have the same underlying mathematics.
1. Compare the tasks and discuss the sources of their difficulty
2. How would you describe each Version: Novice, Apprentice or Expert task?
3. If you are a "math person" estimate the grade at which students (a) Could make some progress (b) Provide a complete solution

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PATCHWORK VERSION 1:

Kate makes patchwork cushions by sewing together right-angled triangles and squares.

She uses triangles along the edges of each cushion, the rest is made from squares.

Kate makes cushions in different sizes.
The picture shows a size 3 cushion.
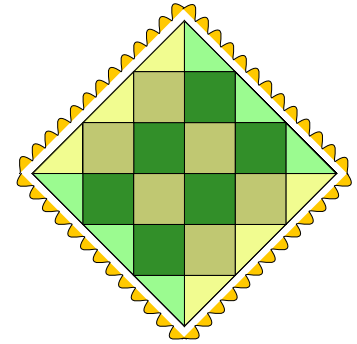
1. Find rules or formulas that will help Kate to find how many squares and triangles she needs for cushions of other different sizes. Explain your work.

2. Kate makes a cushion with 180 squares. What size is it?

PATCHWORK VERSION 2

Variables t and s are defined by the following functions of n

$$t = 4n \qquad\qquad s = 2n(n-1)$$

1. Calculate the values of t and s for $n = 5$

2. If $s = 180$, calculate n

PATCHWORK VERSION 3:

A sheet of square dot paper is provided for use with this item.

Kate makes patchwork cushions.
She uses triangles along the edges of each cushion.
The rest is made from squares.
Here are the first five sizes of patchwork cushions.
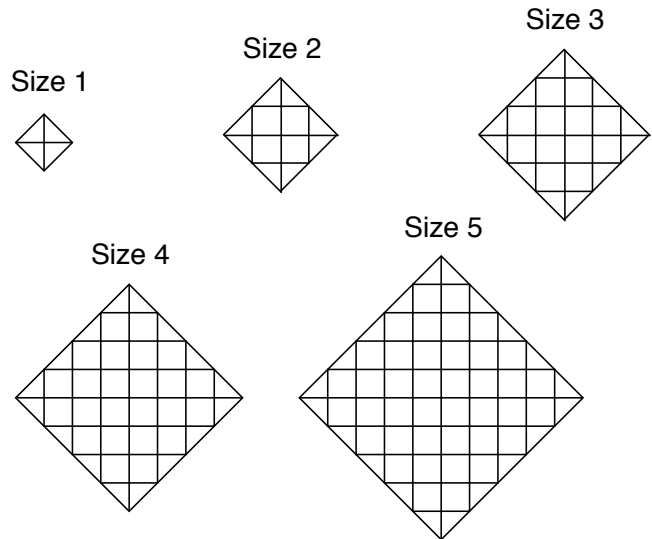
Kate makes cushions in many other different sizes.
She begins to figure out how many triangles and
squares she needs for each size.
For size 1, she needs 4 triangles and 0 squares.
For size 2, she needs 8 triangles and 4 squares.

Size 1

Size 2

Size 3

Size 4

Size 5

1. Complete this table to show how many triangles and squares she needs for each of these sizes.

Size (n)	Number of triangles (t)	Number of squares (s)
1		
2		
3		
4		
5		

2. Find a rule, or a formula, that will help Kate figure out the number of triangles that she needs for cu
   of different sizes. Explain how you figured it out.

3. Use the number patterns in the table to find a rule, or a formula, that will help Kate figure out the
   number of squares she needs for cushions of different sizes. Explain why your rule works.

4. Kate has a cushion made with 180 squares.
   How many triangles are in this cushion?
   Show how you found the number of triangles.

Handout 4: A sample of tasks from our curriculum

These examples show the range of task types included in our curriculum of Grade 7

Curriculum balance
The proportion of our student's class time spent on each type is approximately
Expert: 10%
Apprentice: 20%
Novice 70%

Handout 5: A sample of tasks from our tests

These examples show the range of task types included in our tests for Grade 7

Curriculum balance
The proportion of our student's class time spent on each type is approximately
Expert: 10%
Apprentice: 20%
Novice 70%

Model for Handout 4: A sample of tasks from our curriculum

These tasks include examples of Expert, Novice and Apprentice tasks in our curriculum.

Candy Box

You work for a design company
you've been asked to design a box to hold 18 candies
Each candy is a disc, 2 cm in diameter, 1 cm thick.
Each box must be made from a single sheet of Letter-size card, with as little cutting as possible.
Make two designs for the box and give reasons why one is better than the other.
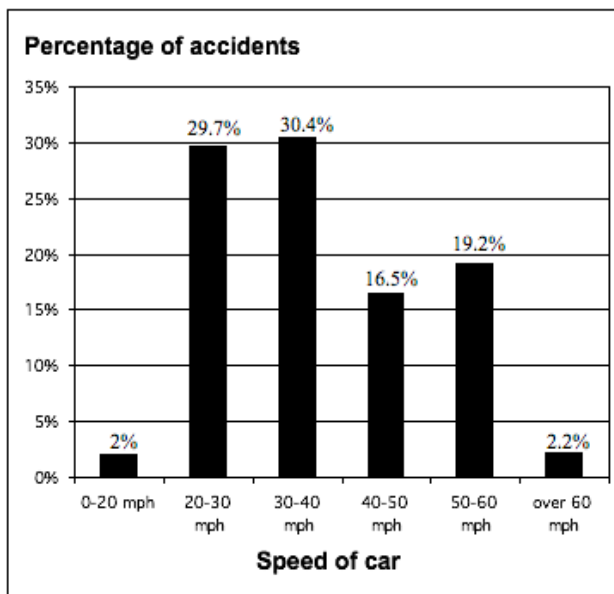
Bulbs

In a shipment of 1,000 bulbs, $\frac{1}{40}$ of the bulbs were defective.

What is the ratio of defective to non-defective bulbs?

Accident Data

The following real data shows how the percentages of cars involved in traffic accidents is related to the speed at which they were driving.

Percentage of accidents

(a) Max claims that the graph shows that almost two thirds of accidents occur to cars travelling below 40mph?
(b) Max concludes that it is safer to drive at over 40 mph than to travel below 40mph.

Is Max right in drawing each of these conclusions? Explain why or why not.

Driving Test

Ann teaches people to drive.
The table below shows how many of her pupils have passed and failed the driving test.

	Pass	Fail
Men	15	5
Women	30	0

1. What percentage of the women passed?
2. What percentage of the men passed?
3. What percentage of the passes were by men?
4. What percentage of all her pupils have passed?

Rectangles and pentagons

A rectangle has more right angles than a pentagon.

This is true: **in all cases / in some cases / in no cases**.
(Please circle the right answer)
Explain and justify your answer below.

Cat Food

Carol has two cats, Rover and Bobo.
Rover eats 3/4 of a can of cat food each day
Bobo eats 1/2 of a can of cat food each day.

Cat food costs $5.00 for three cans.
It is only sold in 3 can packs.

1. How much does it cost Carol for a 60-day supply of cat food for her two cats? $_____
Show your work.

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

29 days $_____
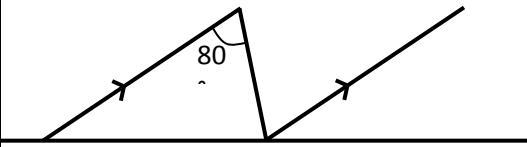30 days $_____
31 days $_____
Show your work.

3. What do you notice about your answers?

Complements

On this diagram, label another angle whose measure is 80°.

Greatest?

Draw a circle around the expression that is greatest when n is a negative number.

$n - 2$ $2n$ n^2

$$\frac{n}{2}$$ $$\frac{2}{n}$$

Fencing

A straight fence is constructed from posts 6 inches wide and separated by lengths of chain 5 feet long. The fence begins and ends with a post.

If there are 7 posts, what is the total length of the fence? (12 inches = 1 foot)

Organizing a tennis tournament

You have the job of organising a table tennis league.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

Plan how to organise the league, so that the tournament will take the shortest possible time. Put all the information on a poster so that the players can easily understand what to do.

Lawn mowing

60 yards
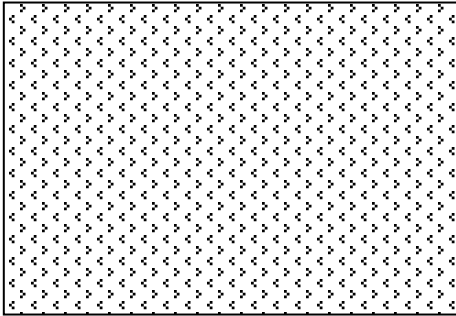
40 yards

Dan and Alan take turns cutting the grass.
Their lawn is 60 yards long and 40 yards wide.

1. What is the area of the yard?
2. Dan takes an hour to cut the lawn using an old mower. How many square yards does Dan cut in a minute? Show your work.

3. Alan only takes 40 minutes using a new mower. How many square yards does Alan cut in a minute? Show your calculation.

4. One day they both cut the grass together.
    How long do they take?
    Show how you figured it out.

Pentagon Problem

*This pentagon has 3 equal sides at the top
and 2 equal sides at the bottom.
3 of the angles have a measure of 130°.*

130°   130°

x°   *x°*
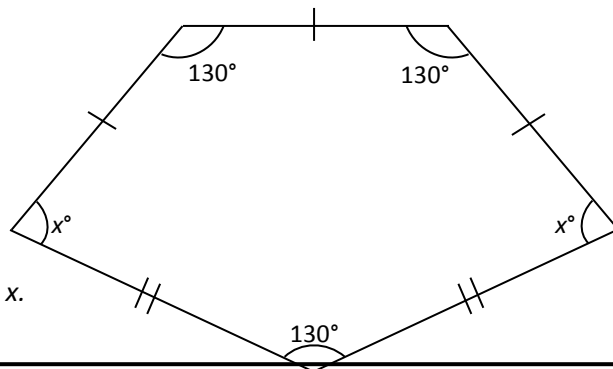
Figure out the measure of the angles marked x.

130°

Find another method of calculating this result.

Strange but True

Many years ago an Australian newspaper reported the following amazing story:

"Mike Dolega has the strangest obsession with numbers. Over the last two years, Mike has written out all the numbers from one to a million.

To reach the magical figure of one million, he used forty exercise books.

He wrote the numbers in 10 columns on each page. There were 26 numbers in each column.

He used 97 ball point pens.

The total number of single digits that Mike has written is 5,888,896"

1. How many numbers did Mike write on each page of his exercise books?

2. How many pages were there in each exercise book?

3. Check that the figure 5,888,896 is correct. Describe your method and show your working in an organised way.

Volume of Soda

Mrs. Grundy is planning to sell her home-made cola.
These pictures show the top and side views of the type of bottle she plans to use.
They are drawn accurately, full size. *(not so here!)*

1. Calculate the volume of soda that is now in the bottle, in cubic centimetres.
   Do this as accurately as you can.
   Show your method clearly. State any formulae that you use.

2. Do you think that your calculation for the volume is too large or too small?
   Explain why you think this.
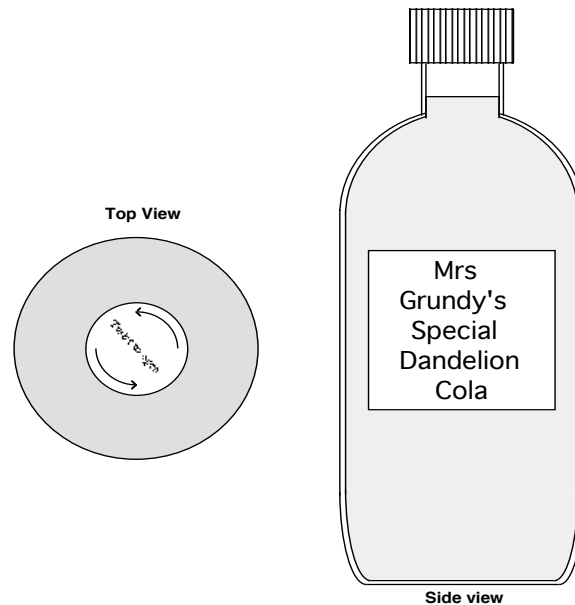
Top View

Mrs Grundy's Special Dandelion Cola

Side view

Curriculum balance
The proportion of our student's class time spent on each type is approximately
Expert: 20%
Apprentice: 20%
Novice: 60%

Suggested classifications of the tasks on Model for Handout 4, with explanation

N = Novice task, E = Expert task, A = Apprentice task. **See Handout 1**.

(Assumes E and A examples are non-routine - i.e. have not been taught and practiced)

Bulbs	N	Routine exercise, taught and practiced
Candy Box	E	Working out how to do it is a major challenge
Accident Data	E	Interpreting graph must be integrated with understanding the situation in (b)
Driving Test	N	Routine
Rectangles and Pentagons	E	Requires investigation and carefully constructing a chain of reasoning
Magic Sum Puzzle	A	Qu1 may be done by trial and improvement; Qu2, 3 are nicely 'ramped'
Cat Food	A/N	A sequence of routine exercises but with some complexity, and an E part 3
Greatest	N	Solvable by trying examples
Fencing	N	Provided you are taught to draw diagrams!
Complements	N	Routine
Organizing a Tennis Tournament	E	Complexity and long chains of reasoning
Lawn Mowing	A	Ramp of complexity; Part 4 has significant formulation challenge
Pentagon Problem	E/N	E at Grade 7 (expert tasks don't need to be difficult) N in high school
Strange but True	A	Note the ramp of difficulty/complexity (Part 3. alone ~ E)
Volume of Soda	E	A realistic combination of measurement, modeling, and using formulas