Mathematics Improvement Network

Program Coherence Health Check

Are we well-aligned?

Workshop Outline

- What is program coherence?
- Key diagnostic probe: the range of task types
- Expert, Apprentice and Novice Tasks
- **Looking at our program**
- Different kinds of math challenge
- What have we learned? - a forward look

What is program coherence?

A coherent mathematics program:

- **is a focused, logical, clearly articulated set of concepts and skills**
- **with connections** to each other and to applications in a variety of contexts from the real world and within mathematics.
- **is uniform across buildings and the district** providing all students the opportunity to learn important mathematics
- **is supported by all aspects of the districts program.**

What are the key "drivers"?

We send messages to math teachers through:

- policy statements
- curriculum materials
- District Assessments
- professional development OFFERINGS

Are they consistent and coherent?

Success in Mathematics

"Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically."

Key diagnostic probe:
The range of task types

Tasks are central to:

- Defining the curriculum
- Focusing the teaching and learning
- Making sure important mathematics is assessed
- Making policy meaningful

Progression?

In Language Arts:

Progress lies in the ability to read and compose

increasingly complex **texts** in

increasingly sophisticated ways

In Mathematics:

Progress lies in the ability to tackle, and investigate

increasingly complex **tasks** in

increasingly sophisticated ways

Doing mathematics involves……

- **Content:**
  concepts and skills
  - **Building a toolkit** of knowledge and techniques and making connected sense of it
- **Practices:**
  problem solving and reasoning:
  - **Selecting and using appropriate tools to tackle problems** that arise, in math and in the world beyond the classroom
  - **Explaining** one's reasoning and conclusions

Because ….

Content and Practices
need
different types of task

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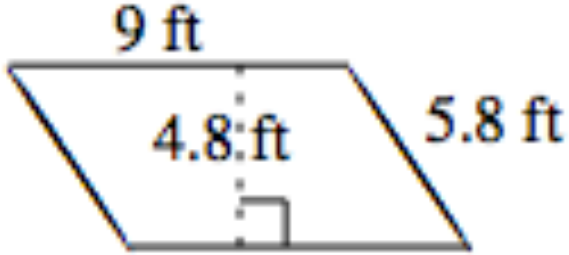
Expert, Apprentice and Novice Tasks

Some examples

Novice tasks

1. $14 \times 32 =$

2. Find the area of

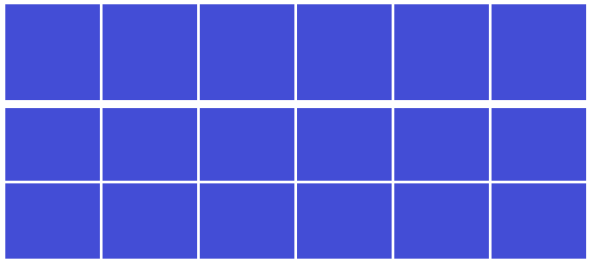


3. Factor the expression: $x^2 + 3x - 4$

4. Write $\sin(A+B)$ in terms of:
   $\sin A$, $\cos A$, $\sin B$ and $\cos B$

A Novice Task

- A rectangular fenced in dog pen has sides of 3 feet, 6 feet, 3 feet, and 6 feet. How many feet of fencing make up the dog pen?



A Novice Task

These three graphs show the functions:

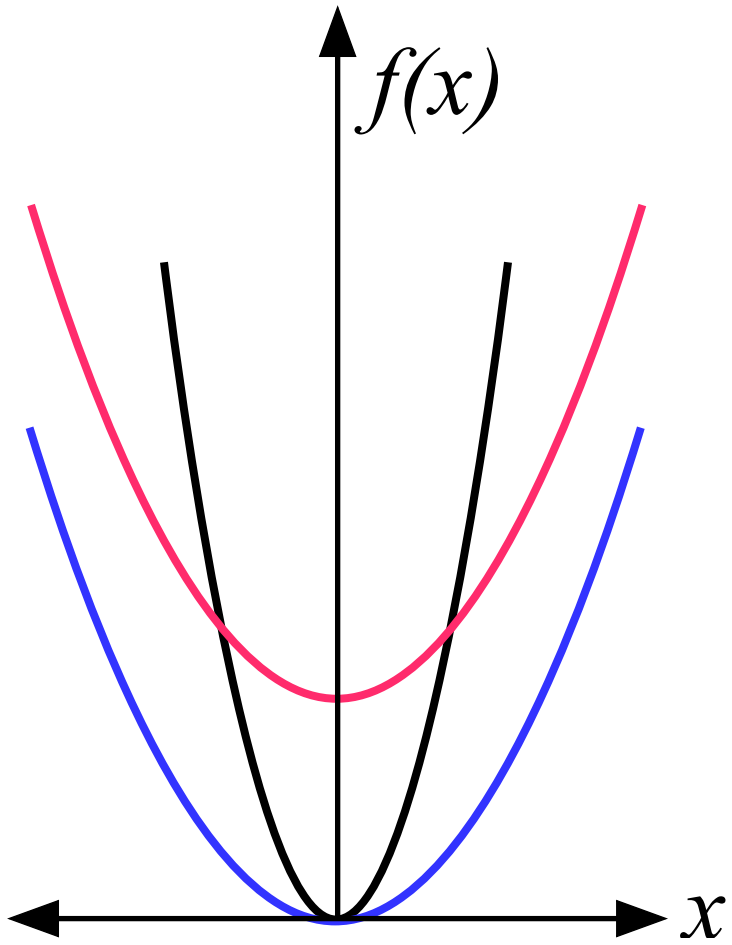
$y = x^2$

$y = x^2 + k$

$y = k\,x^2$

Where: $k > 1$

Label the three graphs



A Novice Task

In the figure to the right, line l is parallel to line *m*. Which of the following pairs of angles must have the same measure?

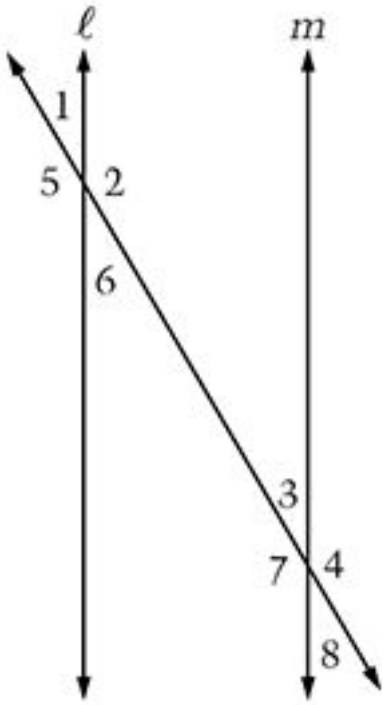
A. Angles 1 and 2

B. Angles 1 and 5

C. Angles 2 and 3

D. Angles 4 and 5

E. Angles 4 and 8



A Novice Task

One of these tables represents a linear relationship, one an exponential growth and one an exponential decay. Label each table correctly.

x	y
1	6
2	9
3	12
4	15

x	y
1	56
2	28
3	14
4	7

x	y
1	6
2	9
3	13.5
4	20.25

Expert tasks

So what do we mean by “expert tasks”?

- Problems, not predigested, but as they arise:
  - in the world outside the math class
  - in really doing mathematics

- Problems in which a taught method and a single skill is not sufficient to complete the task – you have to work out what to do.

Let’s look at a few examples

An Expert Task

Airplane Turn-round

- Between landing and taking off, these jobs all need to be done.
- How quickly can it be done?

	Job	Time needed
A	Get passengers out of the cabin and off the plane	10 minutes
B	Clean the cabin	20 minutes
C	Refuel the plane	40 minutes
D	Unload the baggage from the cargo hold	25 minutes
E	Get new passengers on the plane	25 minutes
F	Load the baggage into the cargo hold	35 minutes
G	Do a final safety check before lift-off	5 minutes

An Expert Task

Traffic Jam

1. Traffic has come to a stop. The line is 12 miles long on a two-lane freeway. How many cars are in the traffic jam?

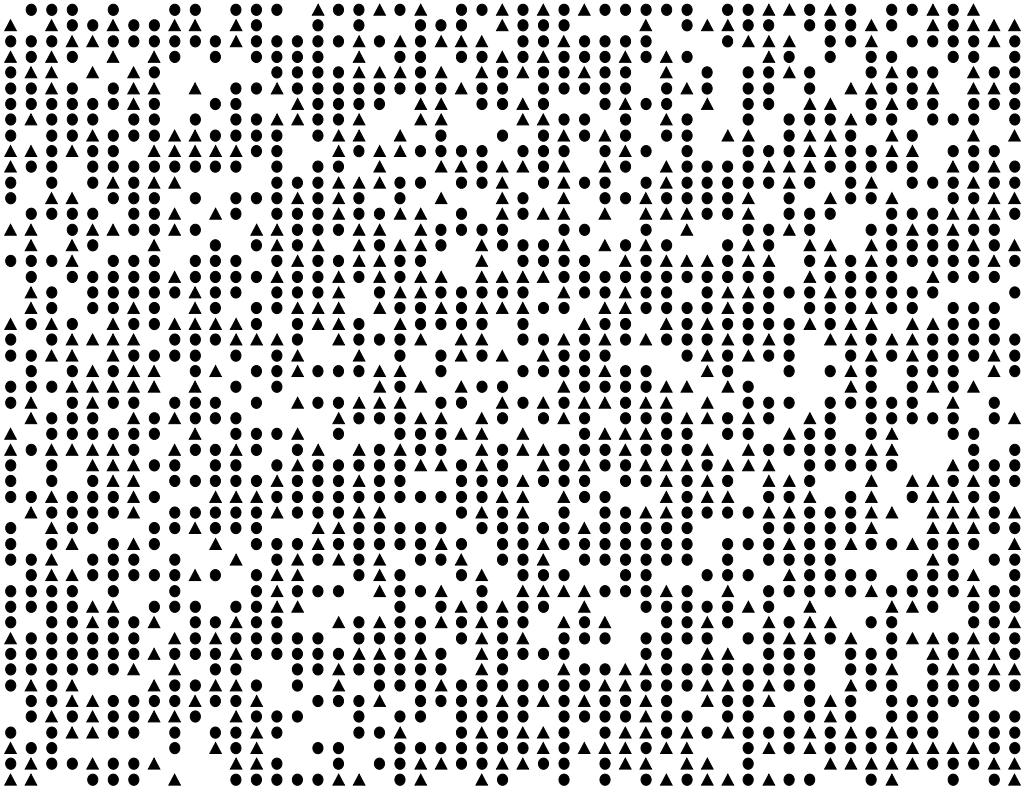
2. Drivers have a two-second reaction time. When the accident was cleared, how long for the last car to move?

An Expert Task

Counting Trees

The diagram shows some trees in a tree farm.

The circles show old trees
The triangles show young trees.



- Think of a method you could use to estimate the number of trees of each type. Explain the method fully.
- Use your method to estimate the number of old trees and young trees.

Apprentice tasks

- Expert tasks with added scaffolding to:
  - ease entry
  - reduce strategic demand
- Ramp of difficulty within the task, with increasing:
  - complexity
  - abstraction
  - demand for explanation
- A step in growing expertise: “climbing with a guide”

An Apprentice Task

I have some pennies, nickels, and dimes in my pocket. I pull out three of the coins and keep them in my hand.

a. How much money do you think I have in my hand? Find as many different amounts as you can think of and list on your paper.

b. Could I have 11¢? How, or why not?

c. Could I have 4¢? How, or why not?

An Apprentice Task

A friend asks you to help them build a rectangular pen for her dog. You have 24 feet of fencing that comes in 1 foot sections.

- Find all the different rectangles you could build. Sketch each one on grid paper.
- Which pen gives the dog the most space to play? Which one gives the dog the least amount of space to play?
- Which would you make for the dog? Why?

An Apprentice Task

Skeleton Towers

How many cubes do you need to make a tower like this:

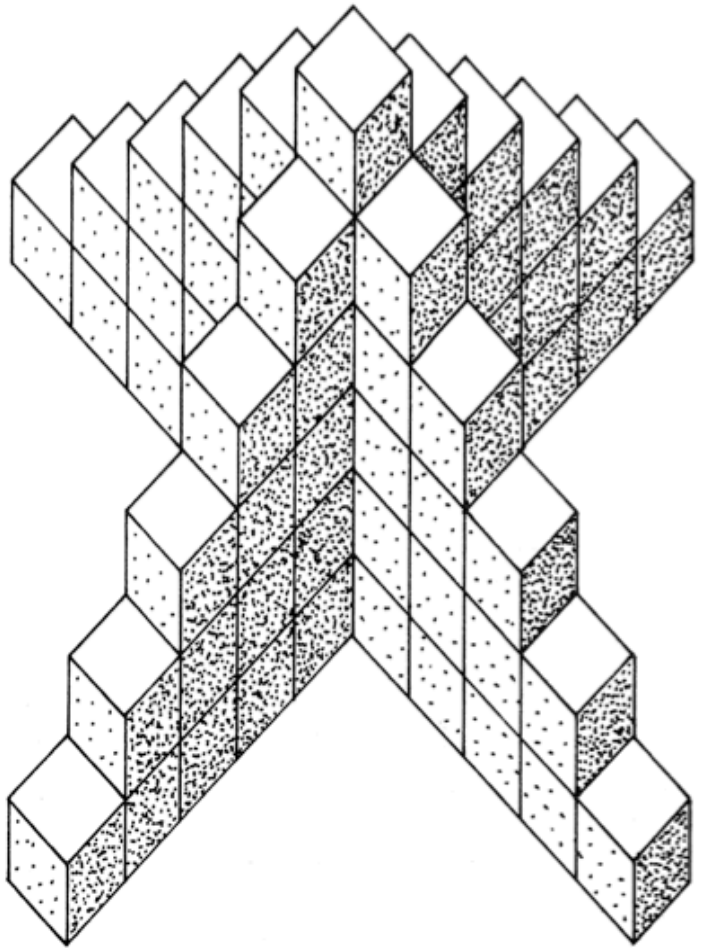
6 cubes high?

20 cubes high?

n cubes high?

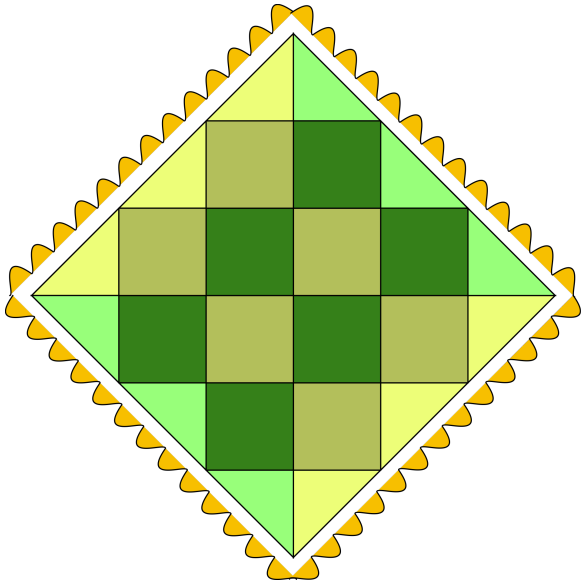
Explain your reasoning

Can you find another method?



Expert, Apprentice or Novice Task?

- **Compare the sources of difficulty:**
  - Complexity?
    Understanding what the task involves
  - Unfamiliarity?
    Working out strategy and tactics
  - Technical demand?
    Selecting the tools to use
  - Autonomy expected?
  - Length of the chain of reasoning?



- *If you are a math person,* **estimate the grade at which students**
  - Could make some progress
  - Provide a complete solution

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Looking at *our* program

Mathematical practices = expert tasks

"Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically."

Does our curriculum develop expertise?

Handout 4 is a sample of the tasks in the curriculum our students go through.

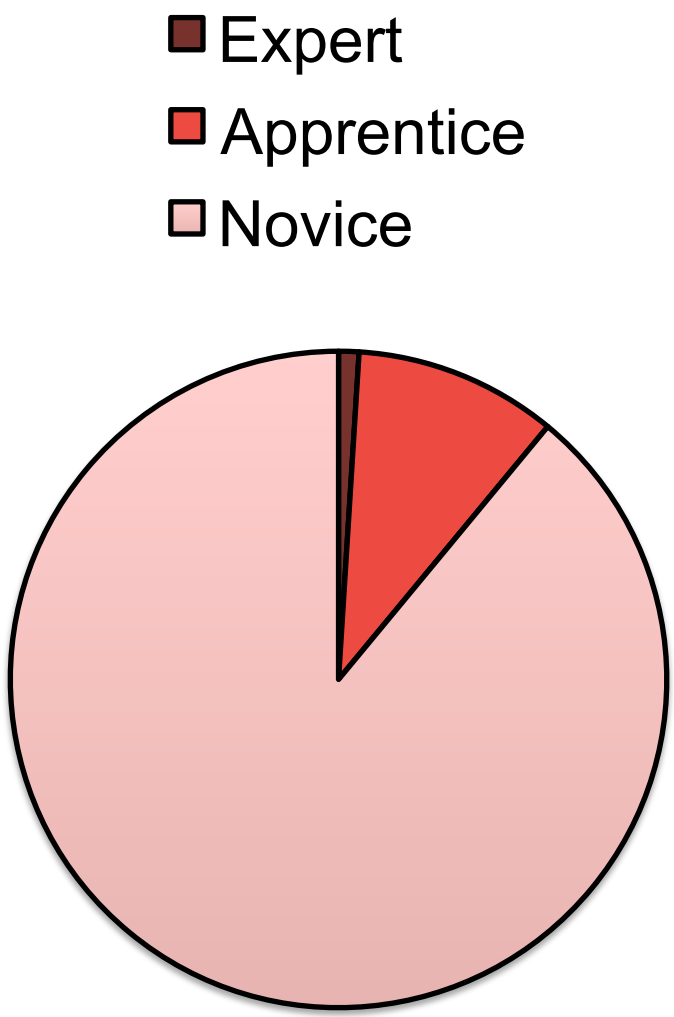
- We have labeled each task E, A or N
  Take a few minutes to review our classifications with your neighbor.

The core questions:

- What is the balance across E, A and N?
- Is it reasonable?

A balanced diet?

- We have looked at our curriculum and estimated the proportions of classroom time that our students spend on Expert, Apprentice and Novice tasks. The numbers are at the end of the Tasks Handout
- Are these proportions all substantial?
- Are they what we want?
- Does the curriculum support teaching for expertise?
- Could teachers ignore teaching for expertise?



Do our tests support expertise?

Handout 5 is a sample of the tasks in the by which we judge our students and teachers.

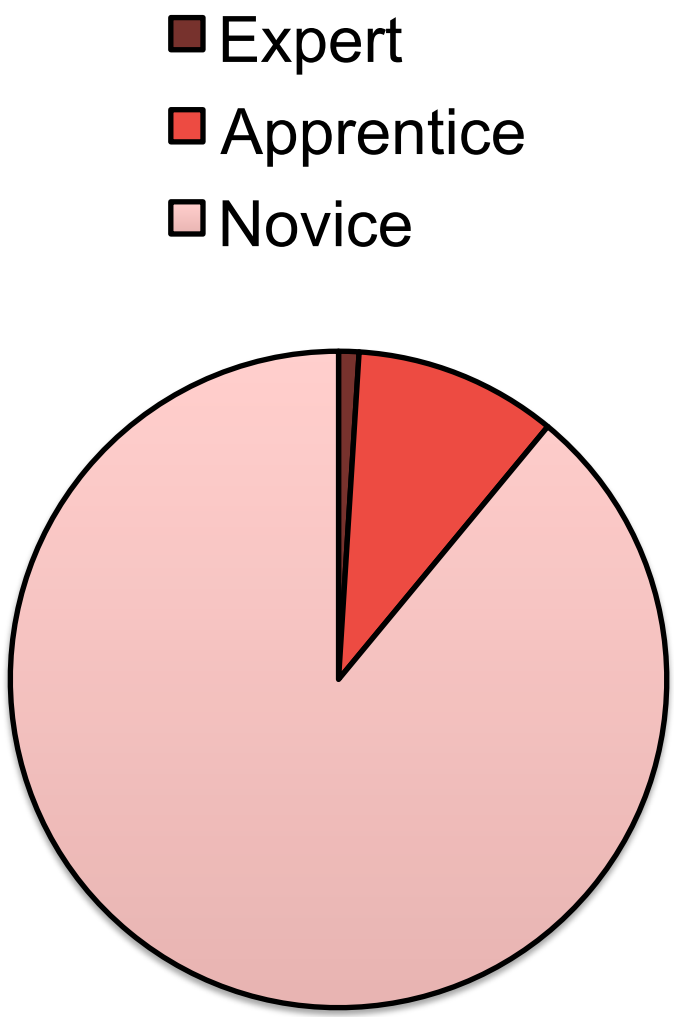
- We have labeled each task E, A or N
  Take a few minutes to review our classifications with your neighbor.

Again, the core questions:

- What is the balance across E, A and N?
- Is it reasonable?

Are our assessments balanced?

- We have looked at our curriculum and estimated the proportions of classroom time that our students spend on Expert, Apprentice and Novice tasks. The numbers are at the end of the Tasks Handout
- Are these proportions all substantial?
- Are they what we want?
- Do the tests encourage teaching for expertise?
- Could teachers ignore teaching for expertise?



Different task types, different roles

- **Content:**
  Concepts and skills can be taught through novice tasks

  Consolidating understanding needs connections = more complex tasks, apprentice and expert

- **Mathematical practices:**
  Problem solving and reasoning can only be developed through working on expert tasks

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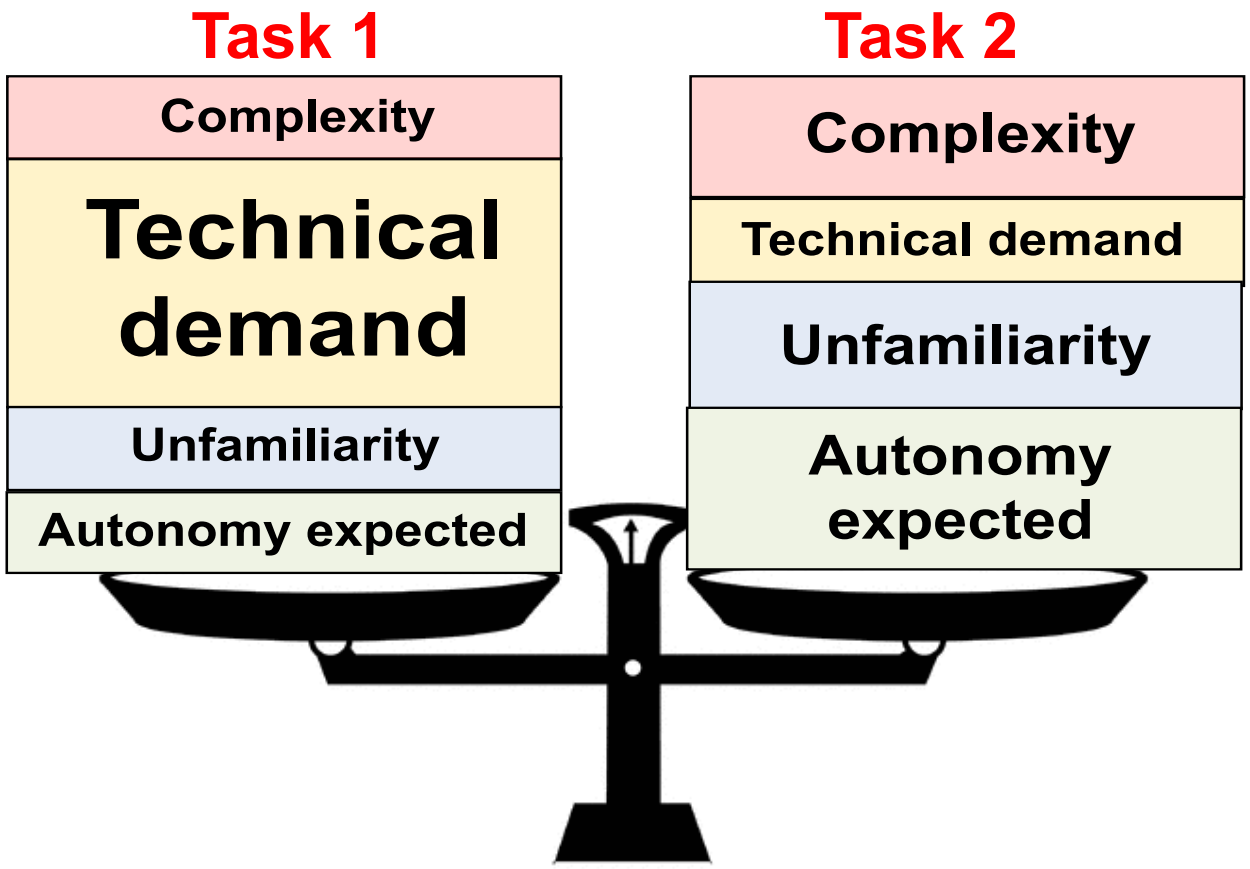
Different kinds of math challenge

Tasks become more difficult as you increase:

- **Complexity**
  - Are there many variables, complex information, messy situations?
- **Technical demand**
  - Are challenging concepts and skills involved?
- **Unfamiliarity**
  - Have students met tasks just like this before?
- **Autonomy expected**
  - Are long chains of unprompted reasoning involved?

Different demands – Similar difficulty



For two tasks with comparable ‘cognitive load’, one that is more demanding in some aspects must be less so in others

Novice	Apprentice	Expert
Complexity **T**echnique **U**nfamiliarity **A**utonomy	**C**omplexity **T**echnique **U**nfamiliarity **A**utonomy	**C**omplexity **T**echnique **U**nfamiliarity **A**utonomy
Short items – skill focused	**Scaffolded tasks**	**Realistic tasks from real world or 'real math'**
Up to grade skills and concepts	**Ramped level of demand**	**Well- absorbed skills and concepts – many from earlier grades**
Liitle from Mathematical Practices	**Modestly involve Mathematical Practices**	**Strongly involve Mathematical Practices**

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What have we learned?

Discussion points

Context

- Many US classrooms have just taught "novice math" – reflecting the short-item tests on which success has been judged. New tests are better.
- Teaching math for expertise is harder, requiring teachers to gain "adaptive expertise" – the ability to modify one's lesson to build on students' responses.

Action

- If our assessment and curriculum are balanced across E, A and N tasks: fine – we can focus on developing teachers' expertise – otherwise,…

Strategies for improvement

- If our testing or our curriculum or both lack substantial components focused on expert and apprentice tasks, we need to consider how we could improve them – now or, better, gradually
- The MathNIC workshop on

"Ways of improving program coherence"

offers a structured discussion of various options for

Improving coherence of curriculum and/or tests

Either way, for success, …..

Ongoing professional development

- It is a truism that teacher expertise is the key to improvement in any education program
- The Common Core (~ international standards) raises the bar for teachers as well as students
- So teacher PD is a key component in making any progress in what happens in classrooms
- Not all types of PD produce changes in teachers teaching.

The MathNIC workshop on

"Designing Professional Development"

will take us through a range of issues and options.

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Thank you

< insert contact details >