Mathematics Improvement Network

# Developing Mathematical Proficiency

## The potential of different types of tasks for student learning

# Workshop Outline

- The Five Strands of Mathematical Proficiency
  - Conceptual Understanding
  - Procedural Fluency
  - Strategic Competence
  - Adaptive Reasoning
  - Productive Disposition
- Introducing the Tasks
- Working on the Tasks
- Feedback on the Tasks
- Reflection

# The Five Strands of Mathematical Proficiency

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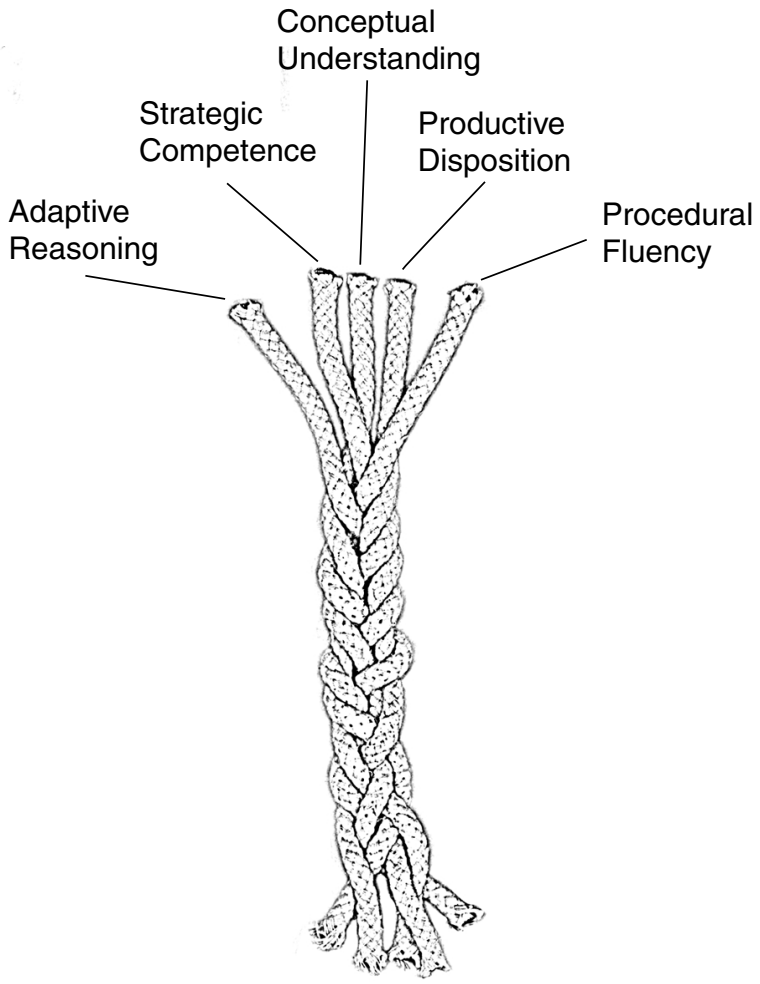
- **Conceptual Understanding**
- **Procedural Fluency**
- **Strategic Competence**
- **Adaptive Reasoning**
- **Productive Disposition**




Kilpatrick, Swafford, & Findell. (2001) – Adding It Up (p.116)

# Developing the Five Strands

| | |
|---|---|
| **Conceptual Understanding** | • Enables students to connect ideas to what they already know<br>• Supports retention and prevents common errors |
| **Procedural Fluency** | • Learning procedures can strengthen and develop mathematical understanding, while understanding makes it easier to learn skills |
| **Strategic Competence** | To come up with answer to a problem, students must:<br>• follow a solution method and adapt as necessary<br>• understand the quantities in the problem and their relationships<br>• represent the relationships mathematically<br>• have the mathematical skills required to solve the problem |
| **Adaptive Reasoning** | As students reason about a problem they can:<br>• build their understanding<br>• carry out the needed computations<br>• apply their knowledge<br>• explain their reasoning to others |
| **Productive Disposition** | Requires frequent opportunities to:<br>• make sense of mathematics<br>• recognize the benefits of perseverance<br>• experience the rewards of sense making in mathematics |

# Conceptual Understanding

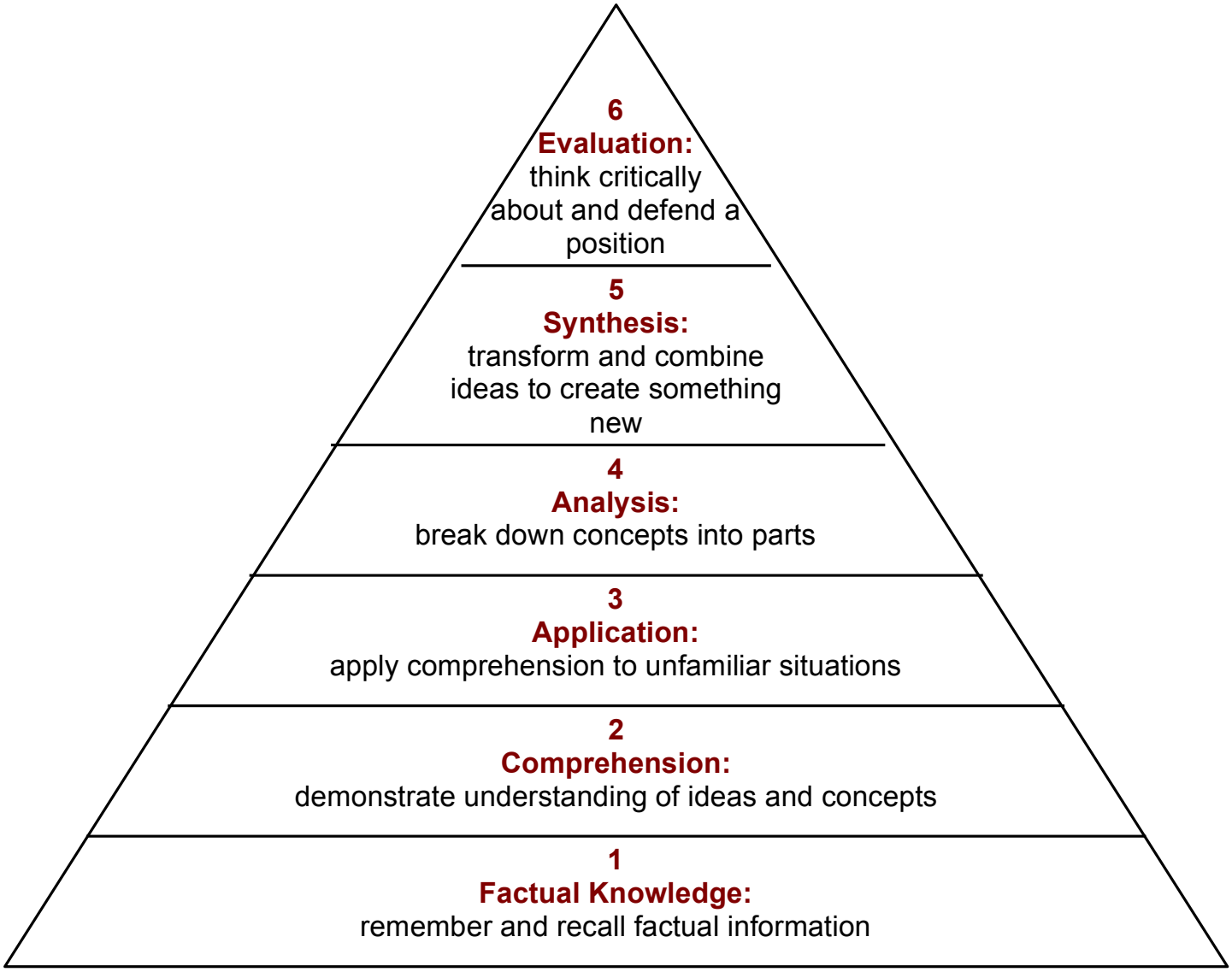
# Conceptual Understanding

"Conceptual understanding frequently results in students having less to learn because they can see the deeper similarities between superficially unrelated situations.

Their understanding has been encapsulated into compact clusters of interrelated facts and principles."

Kilpatrick, Swafford, & Findell. (2001) – Adding It Up (p.120)

# Levels of Conceptual Understanding




Adapted from: https://mathequality.wordpress.com/2012/06/25/nrcs-five-strands-of-mathematical-proficiency/

# Procedural Fluency

# Procedural Fluency

"Business and political leaders are asking schools to ensure that students leave high school 'college and career ready,' possessing 21st Century competencies that will prepare them for adult roles as citizens, employees, managers, parents, volunteers, and entrepreneurs. Using mathematics effectively to solve real-world problems is a critical component of those competencies, and, consequently, is a strong emphasis in the Common Core State Standards for Mathematics and other high-quality standards. Developing procedural fluency is a critical part of instruction to ensure that students are adequately prepared for their futures."

NCTM President Diane Briars (2014) Students Need Procedural Fluency in Mathematics

# Procedural Fluency

The ability to apply appropriate procedures:

- **Accurately** - reliably producing the correct answer
- **Efficiently** – carrying out procedures easily, keeping track of sub-problems, and making use of intermediate results to solve a problem
- **Flexibly** - knowing more than one approach, choosing an appropriate strategy, and using one method to solve and another method to double-check

# Strategic Competence

When faced with a mathematical problem students who are strategically competent have the ability to:

## Formulate

- Identify accessible questions
- Make simplifying assumptions
- Identify significant variables and generate relationships between them

## Represent

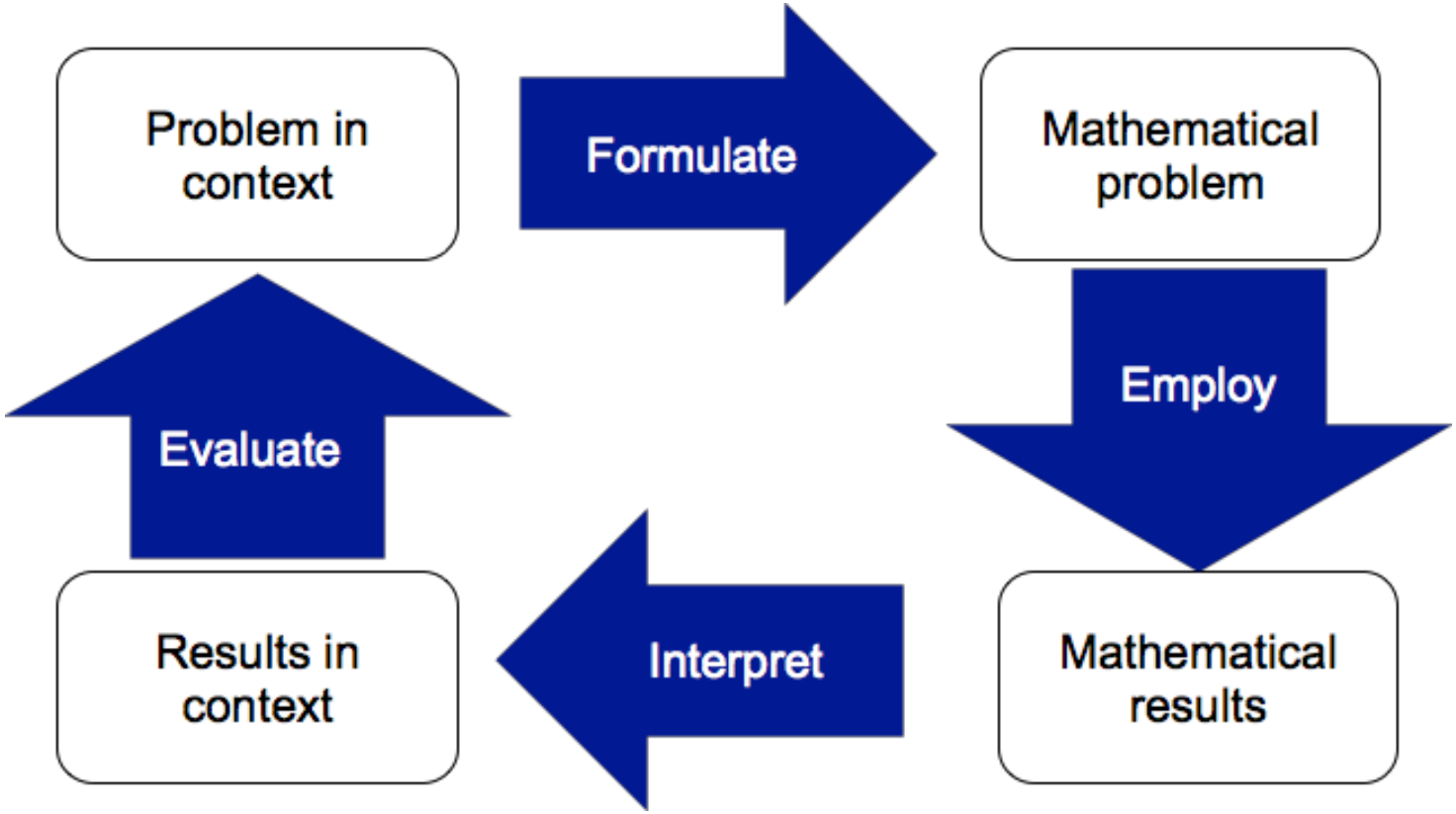
- Represent the situation mathematically, selecting appropriate mathematical concepts and procedures

## Solve

- Monitor progress in a solution approach, changing direction as needed
- Interpret and evaluate results in the context of the problem
- Explain why a conclusion does or doesn't make sense

# Strategic Competence

# Students as Active Problem Solvers




Reproduced from https://www.oecd.org/pisa/pisaproducts/Draft%20PISA%202015%20Mathematics%20Framework%20.pdf

# Relationship Between the Strands

- **Strategic Competence**
- Provides a context for developing conceptual understanding
- Depends on conceptual understanding and procedural fluency

- **Procedural Fluency**
- Develops as strategic competence is used to select procedures

- **Conceptual Understanding**
- Develops as new skills are acquired through increased competence at devising strategies for solving problems

# Adaptive Reasoning

# Adaptive Reasoning

"One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from."

*(Common Core Standards, page 4)*

# Relating the Strands

- **Adaptive reasoning** interacts with the other mathematical proficiency strands

- **Conceptual understanding** provides representations that can serve as a source of adaptive reasoning

- Solution strategies require **procedural fluency** and adaptive reasoning is used to determine the appropriateness of procedures

- **Strategic competence** is used to monitor progress toward a solution and and to generate alternative plans as necessary

# Productive Disposition

# Productive Disposition

- A productive disposition develops with the other strands, helping each of them to develop.
- Students' disposition toward mathematics is a major factor in determining their educational success.
- Students who have developed a productive disposition are confident in their knowledge and ability.

# Introducing the Tasks

# Task A: Percent Change Game

Use these 12 numbers to fill in the gaps below.

**10, 20, 25, 35, 40, 50, 60, 70, 75, 80, 90, 100**

$____ increased by ____% = $____

$____ increased by ____% = $____

$____ decreased by ____% = $____

$____ decreased by ____% = $____

You score a point for each number **used only once** – and in a correct expression. (Maximum score: 12 points)

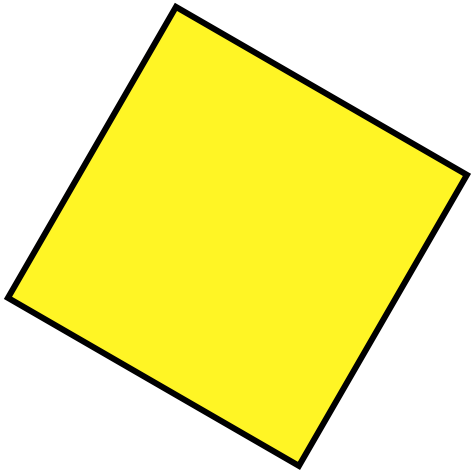
Adapted from http://www.foster77.co.uk/Percentage%20Change%20TASK%20SHEET.pdf

# Task B: Describing and Defining a Square

Four equal sides

Two pairs of parallel sides

Two equal diagonals



Rotational symmetry of order 4

Diagonals meet at right angles

4 lines of symmetry

Four right angles

Which *pairs* of statements *define* a square?
Which pairs do not?

A lesson called 'Describing and Defining Quadrilaterals' based on this task, can be found on the Mathematics Assessment Project website (http://map.mathshell.org/)

# Task C: Always, Sometimes or Never True?

If you add the same number to the top and bottom of a fraction, the fraction increases in value.

If you divide the top and bottom of a fraction by the same number, the fraction gets smaller in value.

If you multiply 12 by a number, the answer will be greater than 12.

If you divide 12 by a number, the answer will be less than 12.

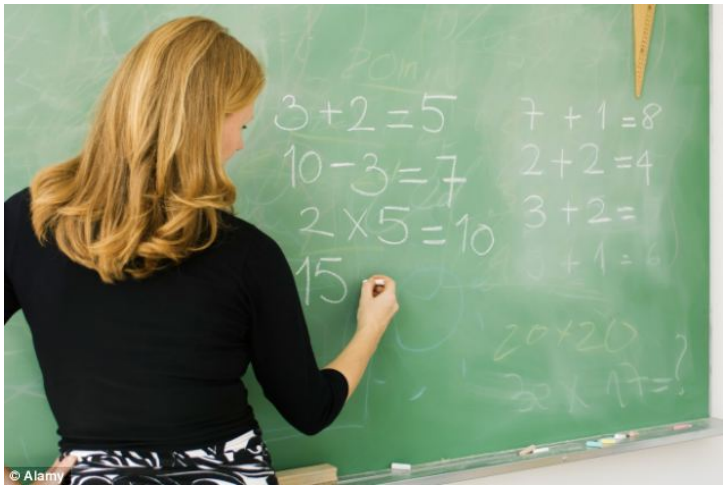
Prices increased by 20%. They then decreased by 20%.
There was no overall change in prices.

Jill got a pay rise of 3%. James got a pay rise of 2%. Jill therefore got the greater pay rise.

# Task D: Schoolteachers and Dentists

There are about 320 million people in the US.

- About how many school teachers are there?




- About how many dentists are there?

Estimate some other facts and check them out.

Adapted from http://www.bowlandmaths.org.uk/pd/pd_01.html

# Working on the Tasks

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- On your own, have a go at the task.
- Discuss in your groups which of the five strands of mathematical proficiency you think your task could support, and how.
- Consider ways in which the task might be modified to address the other strands that it doesn't currently support.
- Elect someone in your group to be the spokesperson for presenting your ideas to the rest of the group.

# Feedback on the Tasks

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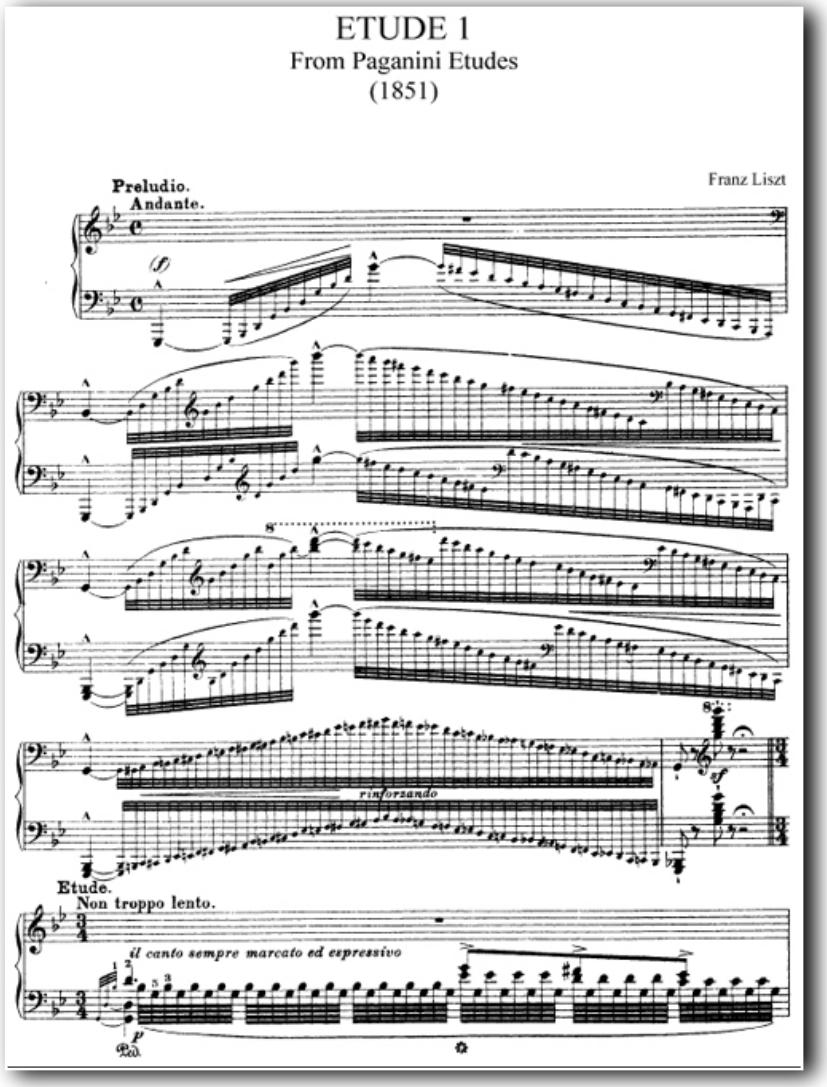
# Études

Études are especially written to practice particular technical skills, but are still beautiful pieces of music in their own right.



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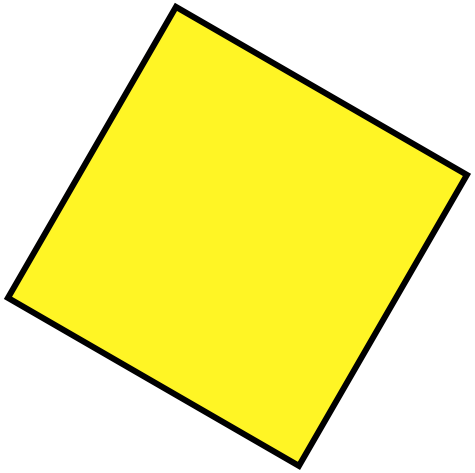

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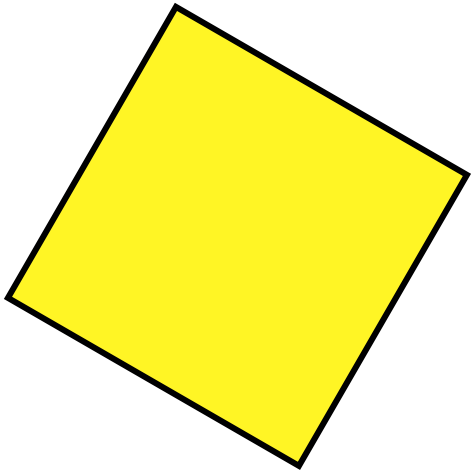

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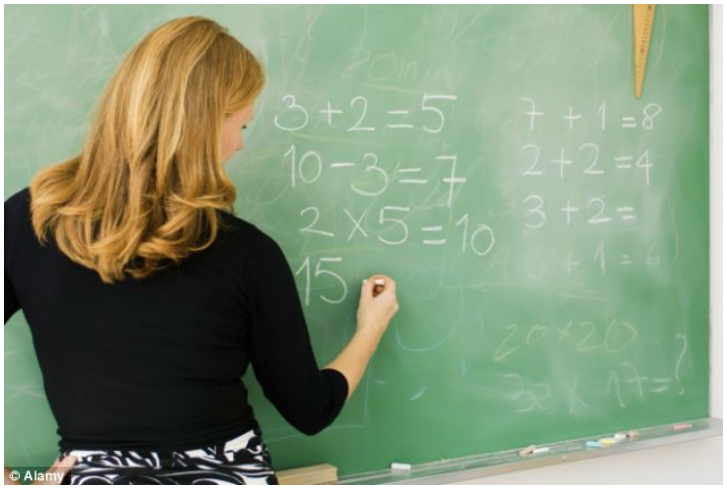


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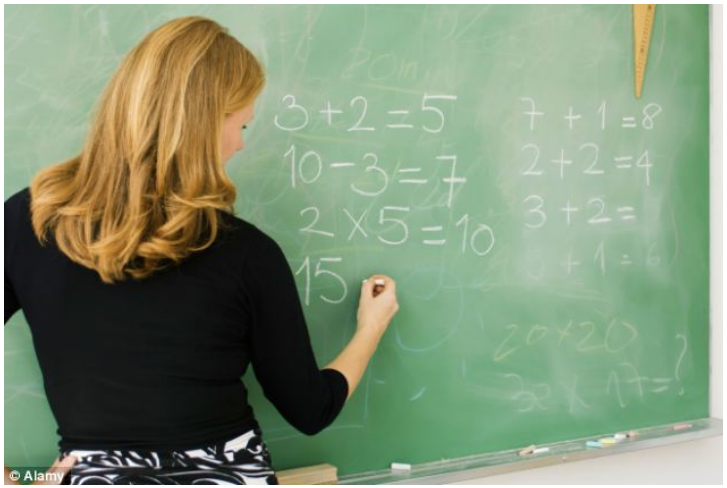


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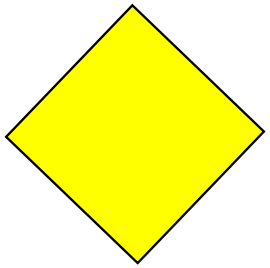
## Describing and Defining a Square



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## Always, Sometimes or Never True?

Here is a collection of mathematical statements or conjectures:

- If you add the same number to the top and bottom of a fraction, the fraction increases in value.
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# Reflection

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- Think about tasks that are already in use (either by you or well known tasks) and categorize them under the five strands of mathematical proficiency.

- Which of the five strands of mathematical proficiency do students currently have the most opportunity to develop in your classroom?

- How do you plan to include a balance of the mathematical proficiency strands in your curriculum over the coming weeks.

Mathematics Improvement Network

# Thank you

**< insert contact details >**